

1.2/1.4 Determining the Number of Elements in Sets

There are two ways to determine the number of elements present:

- Figure it out from a Venn Diagram
- Use a Formula

Either of these methods is acceptable, and you should use the one that you are most comfortable with.

Notation: Suppose we want to determine the number of elements in Set A. We would write this as $n(A)$. If we wanted to determine the number of elements in the union of Sets A and B, we would write this as $n(A \cup B)$.

Example 1

Consider a universal set of all integers from -3 to 3 . Set A is the set of non-negative integers and set B is the set of integers divisible by 2.

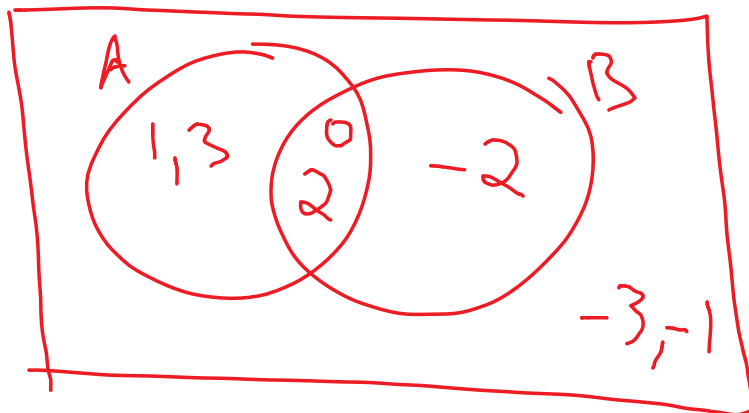
(A) List out the elements found in each set.

$$U = \{-3, -2, -1, 0, 1, 2, 3\}$$

$$A = \{0, 1, 2, 3\}$$

$$B = \{-2, 0, 2\}$$

(B) Create a Venn Diagram and use it to determine $n(A \setminus B)$.



$$n(A \setminus B) = 2$$

(C) Write a formula to represent $n(A \setminus B)$, and use it to calculate $n(A \setminus B)$.

$$n(A \setminus B) = n(A) - n(A \cap B)$$

Principle of Inclusion and Exclusion

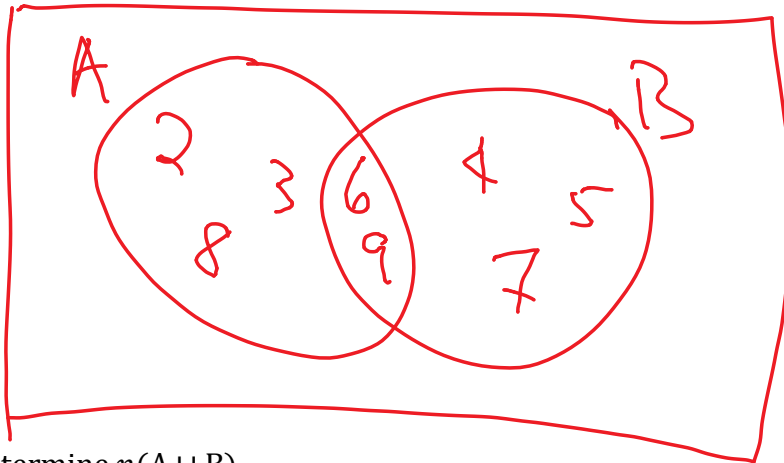
This principle is used to determine the number of elements in the **union** of sets.

Example 2

Set A has elements {2, 3, ~~6~~, 8, 9}

Set B has elements {4, 5, ~~6~~, 7, 9}

(A) Draw a Venn Diagram representing the sets.



(B) Determine $n(A \cup B)$.

$$n(A \cup B) = 8$$

(C) Determine the value of $n(A) + n(B)$.

$$n(A) = 5 \quad n(A) + n(B) = 10$$

$$n(B) = 5$$

(D) Is the value of $n(A) + n(B)$ equal to $n(A \cup B)$? Explain.

NO. We need to subtract the overlap.

When calculating the number of elements in the union of two sets we can use the formula:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

This is called the Principle of Inclusion and Exclusion. What is the value of $n(A \cap B)$ if Sets A and B are disjoint?

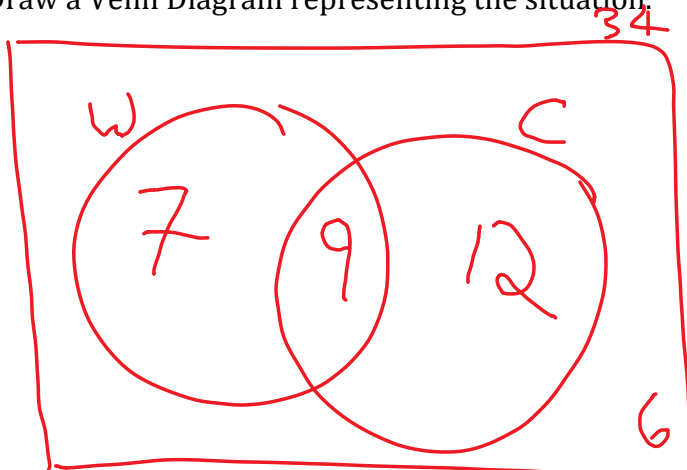


Important Note: When asked to calculate the number of elements in the union of two sets, you can use **either** a Venn Diagram **or** the formula for the Principle of Inclusion and Exclusion. You do **not** need to know how to do both. Use the method you are most comfortable with!

Example 3

Bradley does a survey of 34 people to find out how many people go to a gym each week and what type(s) of training they do. He learned that each week, 16 people do weight training (W), 21 people do cardio training (C), and 9 people do both types of training. 6 people do not go to the gym.

(A) Draw a Venn Diagram representing the situation.



(B) Determine $n(W)$.

$$n(W) = 7 + 9 = 16$$

(C) Determine $n(C)$.

$$n(C) = 9 + 12 = 21$$

(D) Determine $n(W \setminus C)$.

$$n(W \setminus C) = 7$$

(E) Determine $n(W \cup C)$.

$$n(W \cup C) = 7 + 9 + 12 = 28$$

(F) Determine $n(W \cap C)$.

$$n(W \cap C) = 9$$