Math 3201

2.1 The Fundamental Counting Principle

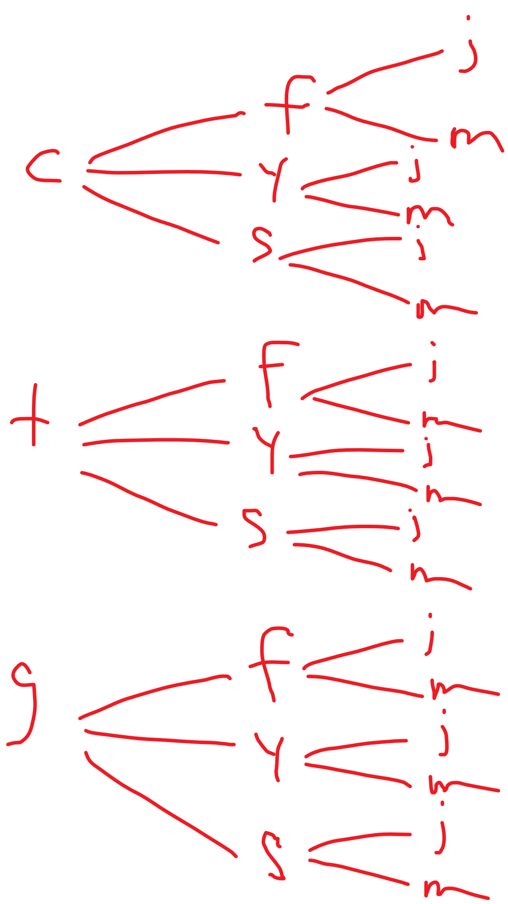
The ***Fundamental Counting Principle*** is a way of determining the number of possible ways that we can perform two or more operations **together**. If operations were being performed independent of each other instead of together, then we would **NOT** use the Fundamental Counting Principle.

In Grade 7, you learned some counting methods, such as using tree diagrams and tables. We will solve the next example using tree diagrams, and then we will learn how to solve it using the fundamental counting principle.

Example 1:
The school cafeteria restaurant offers a lunch combo for $6 where a person can order:

1 sandwich:	chicken, turkey, grilled cheese
1 side:	fruit, yogurt, soup
1 drink:	juice, milk

Draw a tree diagram or create a table to determine the possible lunch combos.



Count up the ends of the "branches": 18

18 combinations

Suppose we added and extra type of sandwich, an extra type of side dish, and an extra type of drink. What impact would this have on our tree diagram? Would it still be feasible to use a tree diagram?

No. Not feasible.

Too messy, too cluttered

Limitation of Tree Diagrams: ok when small # of events.

In cases where there are a large number of operations being performed together, it is not feasible to construct a tree diagram. It is more reasonable to use the **Fundamental Counting Principle** to determine the number of ways of performing all the tasks together.

Fundamental Counting Principle

# of ways of performing tasks together = (# ways to perform 1st task) ×
(# ways to perform 2nd task) ×
(# ways to perform 3rd task) × ...

Let's take a look at the lunch combination example using the fundamental counting principle.

# of possible lunch combos. = (# sandwich types) × (# side dishes) × (# drink types)
$3 \times 3 \times 2$
$= 18$

Note: The Fundamental Counting Principle can only be used when each of the given tasks are being performed together. From our example, there were three tasks: sandwich, side dish and drink. A lunch combination consisted of **ALL** three tasks. That is, it could **NOT** include just a sandwich or just a side dish or just a drink. In cases where a student only bought one item, just a sandwich for example, we would not be able to use the fundamental counting principle. We will discuss this in more detail shortly.

Example 2:

How many variations of ice cream sundaes can be made if the ice cream choices are vanilla and chocolate, the toppings are nuts or candy pieces, and the sauces are chocolate, fudge or strawberry. Each sundae must contain one type of ice cream, one topping, and one sauce. Draw a tree diagram to show how many different types of sundaes can be made and then verify the solution using the Fundamental Counting Principle.

possible sundaes $= 2 \times 2 \times 3 = 12$

Example 3:

Newfoundland & Labrador license plates consists of a **letter letter letter digit digit digit** arrangement, such as CXT 132.

(A) How many license plate arrangements are possible if both letters and digits **CAN** be repeated?

possibilities $= \underline{26} \times \underline{26} \times \underline{26} \times \underline{10} \times \underline{10} \times \underline{10}$

$= 17\ 576\ 000$

(B) How many license plate arrangements are possible if no letter or digit can be repeated?

possibilities $= \underline{26} \times \underline{25} \times \underline{24} \times \underline{10} \times \underline{9} \times \underline{8}$

$= 11\ 232\ 000$

(C) How many license plate arrangements are possible if vowels are not allowed, but repetition is permitted? (a, e, i, o, u)

possibilities $= \underline{21} \times \underline{21} \times \underline{21} \times \underline{10} \times \underline{10} \times \underline{10}$

$= 9\ 261\ 000$

Example 3:
Canadian postal codes consist of a **letter digit letter digit letter digit** arrangement.

(A) How many codes are possible if repetition is permitted?

$$\# \text{ possibility} = \underline{26} \times \underline{10} \times \underline{26} \times \underline{10} \times \underline{26} \times \underline{10}$$
$$= 17\,576\,000$$

(B) In Newfoundland and Labrador, all postal codes begin with the letter A. How many postal codes are possible? Again repetition is permitted?

$$\# \text{ possibilities} = \underline{1} \times \underline{10} \times \underline{26} \times \underline{10} \times \underline{26} \times \underline{10}$$
$$= 676\,000$$

"And" vs. "Or"

Recall the lunch combination example. Suppose student had to order a sandwich AND a side dish AND a drink. In this case we would use **Fundamental Counting Principle**, in which we would **multiply** the number of options in each task together. If there were three types of sandwiches, three types of side dishes and two types of drinks, then the possibilities would be:

$$\# \text{ of possibilities} = 3 \times 3 \times 2 = 18$$

Suppose students were permitted to order a sandwich **or** a side dish **or** a drink. In this case, we **add** the number of options.

Thus the number of possibilities would be :

$$\# \text{ of possibilities} = 3 + 3 + 2 = 8$$

Thus,

· when a problem involves **"and"** - **multiply** the numbers for each task
· when a problem involves **"or"** - **add** the numbers for each task

Example 3:
How many possibly outcomes will there be if you flip a coin and roll a die?

possibilities = $2 \times 6 = 12$

Example 4:
How many possible outcomes will there be if you flip a coin or roll a die?

possibilities = $2 + 6 = 8$

Example 5:
A buffet offers 5 different salads, 10 different entrees, 8 different desserts and 6 different beverages. In how many different ways can you choose a salad, an entree, a dessert, and a beverage?

combos = $5 \times 10 \times 8 \times 6$
$= 2400$

Problems With Restrictions/Conditions

Some of the examples that we did already that did not allow repetition of letters or numbers in a pattern are said to have restrictions. This changes the numbers for the various tasks.

For example, on license plates if repetition was not permitted, there would be 26 possibilities for the first letter, but only 25 for the second, and 24 for the third.

Examples 6:
How many three digit numbers can you make using the digits 1, 2, 3, 4 and 5 if:

(A) repetition of digits is allowed?

possibilities = $\underline{5} \times \underline{5} \times \underline{5} = 125$

(B) repetition of digits is **not** allowed?

possibilities = $\underline{5} \times \underline{4} \times \underline{3} = 60$

Examples 7:
In how many ways can a teacher seat five boys and three girls in a row of eight seats if a girl must be seated at the each end of the row?

(A) Are there any restrictions for seating girls and boys?

yes. A girl at the end of each row.

(B) Why should you fill the girls seats first?

Restriction on where the girls sit.

(C) How many choices are there for seat 1 if a girl must sit in that seat?

3

(D) How many girls remain to sit in seat 8?

2

(E) How many choices of boys and girls remain to sit in each of seats 2 through 7?

6

(F) Which mathematical operation should you use to determine the total number of arrangements?

multiplication

(G) What is the total number of possible outcomes?

$$\underset{G}{\underline{3}} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underset{G}{\underline{2}}$$

$= 4320$

Textbook Questions: page 73 - 75 #2 ,3, 6, 7, 8, 9, 14, 16(a)