Math 3201

2.2A Factorial Notation

Review of the Fundamental Counting Principle

Example 1:
Determine how many possible ways we can arrange the letter A, B and C using the Fundamental Counting Principle.

$\underline{3} \times \underline{2} \times \underline{1} = 6$

Example 2:

(A) In how many different ways can a set of 5 distinct books be arranged on a shelf?

possibilities: $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$
$= 120$

(B) In how many different orders can 15 different people stand in a line?

possibilities: $\underline{15} \times \underline{14} \times \underline{13} \times \underline{12} \ldots \underline{\ }$
$= 1.307674368 \times 10^{12}$

Notice that in each of the previous examples, we ended up taking the total numbers of tasks, and multiplied it by each of the natural numbers below it.

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

Since this often happens in counting problems, it is useful to use something called **Factorial Notation** to speed up the calculations.

The symbol for factorial is $n!$, where n is a natural number. When we take the factorial of a number, we are multiplying the number by all of the natural numbers below it.

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

In general:

$$n! = n(n-1)(n-2)(n-3) \ldots (3)(2)(1), \quad n \in \mathrm{N}$$

ie $10! = 10(9)(8)\ldots(3)(2)(1)$

Example 3:
Evaluate the following:

(A) 8!

$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \cdot 1$
$=$

or

$8! = 40320$

(B) 7! $= 5040$

(C) 10!

$= 3,628,800$

Instead of us writing all of this out and doing the multiplication by hand, we can use the factorial button on our calculators. Find it now and redo Example 3.

Simplifying Expressions Using Factorials

Example 1:
Simplify the following:

(A) $\frac{8!}{5!} = \frac{40320}{120}$

$= 336$

$\frac{8!}{5!} = \frac{8 \times 7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$

$\frac{8!}{5!} = 8 \cdot 7 \cdot 6 = 336$

(B) $\frac{9!}{6!} = \frac{9 \cdot 8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!}}$

$= 504$

(C) $\frac{12!}{9!3!} = \frac{12 \cdot 11 \cdot 10 \cdot \cancel{9!}}{\cancel{9!}\, 3!}$

$= \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1}$

$= 220$

Example 2:
Simplify the following, where $n \in$ I:

(A) $\frac{n!}{(n-2)!}$

$= \frac{n(n-1)\cancel{(n-2)}\cancel{(n-3)}\cdots\cancel{(3)}\cancel{(2)}\cancel{(1)}}{\cancel{(n-2)}\cancel{(n-3)}\cdots\cancel{(3)}\cancel{(2)}\cancel{(1)}}$

$= n(n-1)$

$= n^2 - n$

$n=10$ $(11)(10!) = 11!$

(B) $\frac{(n+1)!}{(n-1)!}$ $= \frac{(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}}$

$= (n+1)n$

$= n^2 + n$

(C) $\frac{(n-3)!}{(n-2)!} = \frac{\cancel{(n-3)!}}{(n-2)\cancel{(n-3)!}}$

$= \frac{1}{n-2}$

(D) $(n+1)n!$

$= (n+1)!$

(E) $(n+3)(n+2)!$

$= (n+3)!$

(F) $(n+1)(n)(n-2)!$

$= \frac{(n+1)(n)(n-1)(n-2)!}{(n-1)}$

$= \frac{(n+1)!}{n-1}$

Textbook Questions: pages 81, 82; 1, 3, 4, 5, 6