Math 3201

2.2B Solving Equations Involving Factorials

Steps:

1. Simplify the side of the equation that contains the factorial notation.

2. Solve the resulting equation. We will only have to deal with linear and quadratic equations in Math 3201.

3. Check for extraneous roots. Recall that we can only take the factorial of natural numbers. In other words n cannot be 0, negative or a fraction. ~~Substitute the n values back into the original equation~~. If any result in the factorial of a non-natural number, then that n value is an extraneous root and we reject them.

Example 1

Solve the following equations, where $n \in$ I.

(A) $\frac{n!}{(n-2)!} = 20$

$(-4)!$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20$$

$$n(n-1) = 20$$

$$n^2 - n = 20$$

$$n^2 - n - 20 = 0$$

$$(n+4)(n-5) = 0$$

$$n+4=0, \quad n-5=0$$

$$\cancel{n=-4}, \quad \boxed{n=5}$$

20
1, 20
2, 10
4, 5

(B) $\frac{(n+2)!}{(n+1)!} = 10$

$$\frac{(n+2)\cancel{(n+1)!}}{\cancel{(n+1)!}} = 10$$

$$n+2 = 10$$

$$n = 10-2$$

$$\boxed{n = 8} \checkmark$$

(C) $\frac{2(n+3)!}{(n+1)!} = 180$

$$\frac{2(n+3)(n+2)\cancel{(n+1)!}}{\cancel{(n+1)!}} = 180$$

$$\frac{\cancel{2}(n+3)(n+2)}{\cancel{2}} = \frac{180}{2}$$

$$(n+3)(n+2) = 90$$

or FOIL

First Outer Inner Last

$$n(n+2)+3(n+2) = 90$$

$$n^2+2n+3n+6-90 = 0$$

$$n^2+5n-84 = 0$$

84

1,84 4,21
2,42 6,14
3,28 7,12

$$(n-7)(n+12) = 0$$

$$n-7=0 \qquad n-12=0$$

$$\boxed{n=7} \qquad n \cancel{=} -12$$

Example 2:
Solve the following equations, where $n \in I$.

(A) $\frac{(n+5)!}{(n+3)!} = 56$

$$\frac{(n+5)(n+4)\cancel{(n+3)!}}{\cancel{(n+3)!}} = 56$$

$$(n+5)(n+4) = 56$$

$$n^2 + 4n + 5n + 20 - 56 = 0$$

$$n^2 + 9n - 36 = 0$$

$$(n-3)(n+12) = 0$$

$$n - 3 = 0 \quad | \quad n + 12 = 0$$

$$\boxed{n = 3} \quad | \quad \cancel{n = -12}$$

36
1, 36
2, 18
3, 12
4, 9
6, 6

(B) $\frac{(n)!}{(n-2)!} = 182$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 182$$

$$n^2 - n - 182 = 0$$

$$(n+13)(n-14) = 0$$

$$n + 13 = 0 \quad n - 14 = 0$$

$$\cancel{n = -13} \quad \boxed{n = 14}$$

182
13, 14

Textbook questions: page 82; 11