Math 3201

2.3B Permutations When Not all Elements are Being Used

We saw in the previous lesson that there are two cases for permutations:

· permutations of n different elements taken n at time
· permutations of n different elements taken r at a time

We also saw that for the first case, when we use **all** of the elements that are available to us, we can simply use the fundamental counting principle to solve problems. In this lesson, we will shift our focus to the second case, when we only use some of the elements that are available to us. To solve these problems, we will need to learn a permutation formula.

Permutation Formula

$$_nP_r = \frac{n!}{(n-r)!}$$

where n is the total number of elements available to us and r is the number of elements that we use at a time.

We normally use this formula when we are **not** using all of the elements available to us, thus, $0 \leq r \leq n$. A good example of this is the problem that supposes we have the digits 1, 2, 3, 4, 5 available to us, and we want to determine how many 3 digit PIN's we can come up with.

We said earlier that when we use all the elements available to us, we can simply use the **Fundamental Counting Principle** to solve problems. However, we can also use the permutation formula in this case as well, as long as $r = n$. In that case, we would get:

$$_nP_r = \frac{n!}{(n-r)!}$$

$$_nP_n = \frac{n!}{(n-n)!} \qquad \text{since } r = n$$

$$_nP_n = \frac{n!}{(0)!}$$

$$_nP_n = \frac{n!}{1}, \qquad \text{since } 0! = 1$$

How many ways can you choose nothing? See page 87 of textbook for explanation.

$$_nP_n = n! \qquad \text{same as Fundamental Counting Principle}$$

Summary

When we use ALL elements available to us, we use the following formula to determine the number of permutations:

$$_{n}P_{n} = n! \qquad \text{same as the Fundamental Counting Principle}$$

When we only use some of the elements that are available to us, we use the following formula to determine the number of permutations:

$$_{n}P_{r} = \frac{n!}{(n-r)!}, 0 \leq r \leq n$$

Example 1:
Shannon downloaded 10 new songs from iTunes. She wants to create a playlist using 6 of the songs, arranged in any order. How many different 6 song playlists can she possibly create?

$n = 10$
$r = 6$

$$_{10}P_{6} = \frac{10!}{(10-6)!} = \frac{10!}{4!} = \frac{10\cdot9\cdot8\cdot7\cdot6\cdot5\cdot\cancel{4!}}{\cancel{4!}} = 151,200$$

or

$$P = \underline{10} \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} = 151,200$$

Example 2:
Lorna has four different colored blocks. Determine how many different groups of two blocks she can come up with.

$n = 4$
$r = 2$

$$_{4}P_{2} = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4\cdot3\cdot\cancel{2!}}{\cancel{2!}} = 12$$

or

$$P = \underline{4} \times \underline{3} = 12$$

Practice:

1. There are 10 movies playing at a theatre. In how many ways can you see two of them consecutively?

$n = 10$

$r = 2$

$_{10}P_2 = \frac{10!}{(8-2)!} = \frac{10!}{8!} = \frac{10 \cdot 9 \cdot \cancel{8!}}{\cancel{8!}} = 10 \cdot 9 = 90$

or

$P = \underline{10} \times \underline{9} = 90$

2. A soccer league has 12 teams, and each team plays each other twice; once at home, and once away. How many games are scheduled?

$n = 12$

$r = 2$

$2 \cdot {}_{12}P_2 = 2 \cdot \frac{12!}{(12-2)!} = 2 \cdot \frac{12!}{10!} = 2 \cdot \frac{12 \cdot 11 \cdot \cancel{10!}}{\cancel{10!}} \quad 2 \cdot 132 = 264$

or

$2 \cdot P = 2 \cdot \underline{12} \times \underline{11} = 2 \cdot 132 = 264$

Textbook Questions: page 93, 94; #1, 3, 5, 8