**Math 3201**

# 2.3C Permutation Problems with Repetition of Elements Allowed

## Review of Permutations Problems

Summary - Permutation Problems Two Methods

$P = \_ \times \_ \times \_ \times \_$

- Draw blanks for each element and write in the number of possibilities for each.
- Use formulas.

**Formulas** (if repetition is **not** permitted):

1. If ALL the available elements are being used: $P = n!$
2. If NOT all of the elements are being used: ${}_nP_r = \frac{n!}{(n-r)!}$

**Rules**

If the repetition of elements is permitted:

· Use the **Fundamental Counting Principle**

If the repetition of elements is **not** permitted:

· Use the permutation formula ${}_nP_r = \frac{n!}{(n-r)!}$

**Example 1:**

A social insurance number (SIN) in Canada consists of a nine digit number that uses the digits 0 to 9. If there are no restrictions on the digits selected for each position in the number, how many SINs can be created if:

(A) No repetition of digits is permitted.

$P = \underline{10} \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} = 3\,628\,800$

${}_nP_r = {}_{10}P_9 = \frac{10!}{(10-9)!} = \frac{10!}{1!} = 3\,628\,800$

(B) Repetition is permitted.

$P = \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10}$

$= 1{,}000{,}000{,}000$

$P = 10^9 = 1{,}000{,}000{,}000$

**Repetition Problems**

When repetition of elements is permitted, we can calculate the number of permutations using the **Fundamental Counting Principle**, which essentially takes the following format:

$$P = n^r$$

**Example 2:**
Suppose we want to create a four digit PIN using the digits 0 - 6. How many different PINs can be generated if:

(A) repetition of digits is **not** permitted.

$P = \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} = 840$

or $_7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 840$

(B) repetition of digits is permitted.

$P = 7^4 = 2401$ or $P = \underline{7} \times \underline{7} \times \underline{7} \times \underline{7} = 2401$

**Examle2:**
Consider the arrangement of a 5-digit password if only the digits 0-9 can be used.

(A) Is the order of the digits in a password important? Explain.

Yes. The wrong order doesn't work.

(B) How many arrangements are possible if repetition is allowed?

$P = \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} \times \underline{10} = 100,000$

Or $P = 10^5 = 100,000$

(C) How many arrangements are possible if repetition is not allowed?

$P = \underline{10} \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} = 30240$

$_{10}P_5 = 30240$

(D) Do the number of choices stay the same if repetition is allowed?

No.

(E) In which case is there a greater number of permutations possible?

Repetition.

**Example 3:**
The code for a lock consists of three numbers selected from 0, 1, 2, 3, with no repeats. For example, the code 121 would not be allowed but 302 would be allowed. Determine the number of possible codes.

$$P = \underline{4} \times \underline{3} \times \underline{2} = 24$$

or

$n = 4$
$r = 3$

$${}_4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = \frac{4 \cdot 3 \cdot 2 \cdot \cancel{1!}}{\cancel{1!}} = 24$$

**Example 4:**
How many two digit numbers can be formed using the digits 1, 2, 3, 4, 5, 6 if repetition is allowed?

$$P = \underline{6} \times \underline{6} = 36$$

$n = 6$
$r = 2$

$$P = n^r = 6^2 = 36$$

**Example 5:**
How many arrangements of three letters can be formed using the letters of the word LOCKERS if repetition is **not** permitted?

$$P = \underline{7} \times \underline{6} \times \underline{5} = 210$$

or

$n = 7$
$r = 3$

$$_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 210$$

**Example 6:**
How many 3 letter arrangements can be formed using the letters M, A, T allowing for repetition of the letters?

$$P = \underline{3} \times \underline{3} \times \underline{3} = 27$$

or

$$P = 3^3 = 27$$

**Textbook Questions:** page 94; #13, 14