Math 3201

2.3D1 Permutation Problems Involving Conditions

Permutation problems sometimes involve **conditions**. For example, in certain situations, objects may be arranged in a line where two or more objects must be placed together, or certain objects must be placed in certain positions.

We will look specifically at three types of conditions:

Case 1: When an element must be placed in a specific position. For example, Morgan must be the first student in the lineup.

Case 2: When two or more elements must be grouped together. For example Michelle and Alicia must sit together in class.

Case 3: When two or more elements must **not** be grouped together. For example, the red book and the blue book must not be placed next to each other on the shelf.

Case 1: **When An Element Must Be Placed in a Specific Position**

We have already did problems like this. We solved by drawing a diagram and using the **Fundamental Counting Principle**. While there are other ways that we can work these out, the method we already used is probably the easiest, so we will continue to go with that.

Example 1:
At a used car lot, seven different car models are to be parked close to the road for easy viewing. The three red cars must be parked so there is a red car at each end, and one exactly in the middle. How many ways could the seven cars be parked?

$$P = \underset{R}{\underline{3}} \times \underline{4} \times \underline{3} \times \underset{R}{\underline{2}} \times \underline{2} \times \underline{1} \times \underset{R}{\underline{1}} = 144$$

* take care of conditions first!!!

Example 2:
Vanessa has a blue book, a red book, a yellow book and a green book that she wants to put in line on a shelf. If the red book must go at either end, how many possible arrangements are there?

$$P = \underline{3} \times \underline{2} \times \underline{1} \times \underset{R}{\underline{1}} = 6$$

or

+

$$P = \underset{R}{\underline{1}} \times \underline{3} \times \underline{2} \times \underline{1} = \underline{6}$$

$$12$$

Case 2: When Two or More Elements Must Be Grouped Together

We will also look at cases in which two elements are to be grouped together. For example, how many arrangements of the word MATH exist if T and H must always be kept together.

Homework

Steps:

1. Treat any elements that must be grouped together as a single element.

2. Determine the total number of groups you have, n, and determine the number of ways they can be arranged, $n!$, using the **Fundamental Counting Principle**.

3. Look at the group that contains more than one element. Call the number of elements in that group e. Figure out how many ways the letters can be arranged in that group $e!$.

4. Since there are ***two conditions*** that must both be met this is an "AND" situation. Thus we must multiply $n!$ and $e!$. For example, there will be a certain number of ways the groups themselves can be arranged, and a certain number of ways the elements within the large group can be arranged.

5. Multiply the number of arrangements for the two conditions:

$$\text{\# of possible arrangements} = n! \times e!$$

Example 3:
How many arrangements of the word FAMILY exist if A and L must always be together?

$$P = \overset{5}{\underset{e}{\boxed{\underline{2} \times \underline{1}}}} \times \underline{4} \times \underset{n}{\underline{3}} \times \underline{2} \times \underline{1}$$

$$P = e! \cdot n! = 2! \cdot 5! = 240$$

Example 4:
At a used car lot, seven different car models are to be parked close to the road for easy viewing. The three red cars must be parked side by side. How many ways can the seven cars be parked?

$$P = \overset{5}{\boxed{\underset{R}{\underline{3}} \times \underset{R}{\underline{2}} \times \underset{R}{\underline{1}}}} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 3! \cdot 5! = 720$$

Case 3: When Two or More Elements Must Not Be Grouped Together

We will not calculate the number of arrangements in these problems directly, but rather we will use the following:

Number of arrangements when two elements **cannot be grouped together**	=	Total number of possible arrangements	−	Number of possible arrangements when the two arrangements **are** grouped together

This is called **indirect reasoning.**

Example 5:
Michael, Bradley, Jarod and Andrew are to be arranged in a line from left to right. How many ways can they be arranged if Michael and Bradley are **not** to stand next to each other?

$$P(\text{all arrangments}) = \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 24$$

$$P(\text{together}) = \boxed{\underset{M}{\underline{2}} \times \underset{B}{\underline{1}}} \overset{3}{\times} \underline{2} \times \underline{1} = 2! \cdot 3! = 12$$

$$P(\text{not together}) = P(\text{all}) - P(\text{together})$$
$$= 24 - 12$$
$$= 12$$

Example 6:
Todd, Jean, Kyle, Colin and Lori are to be arranged in a line from left to right.

(A) How many ways can they be arranged?

$P = \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$

(B) How many ways can they be arranged if Jean and Lori cannot be side by side?

$P(S \times S) = \overbrace{\boxed{\underset{J}{\underline{2}} \times \underset{L}{\underline{1}}}}^{4} \times \underline{3} \times \underline{2} \times \underline{1} = 2! \cdot 4! = 48$

$P(S \times S)' = 120 - 48 = 72$

(C) How many ways can they be arranged if Kyle and Colin must be side by side?

$P = \overbrace{\boxed{\underset{K}{\underline{2}} \times \underset{C}{\underline{1}}}}^{4} \times \underline{3} \times \underline{2} \times \underline{1} = 48$

(D) How many ways can they be arranged if Jean must be at one end of the line? HINT: Jean must be in the first OR the last location.

P(Beginning) or P(end)

$P = \underset{J}{\underline{1}} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} + \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underset{J}{\underline{1}} = 24 + 24 = 48$

Textbook Questions: page 93, 94; #9, 10