Math 3201

2.3E Solving Permutation Equations for n

We will only solve permutation equations for n. We will **not** solve for r.

Steps:

1. The r value will be given, so we must substitute it into the permutation formula.
2. Simplify the side of the equation containing the factorials.
3. Solve the resulting equation for n.
4. Check for extraneous roots. Recall that any n value that results in the factorial of a non-natural number is considered to be an extraneous root.

Examples 1:
Solve the following for n.

$${}_nP_r = \frac{n!}{(n-r)!}$$

(A) ${}_nP_2 = 30$

$$\frac{n!}{(n-2)!} = 30$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 30$$

$$n^2 - n = 30$$

$$n^2 - n - 30 = 0$$

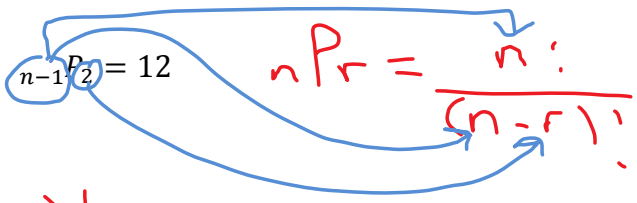
30
1, 30
2, 15
3, 10
5, 6

$$(n+5)(n-6) = 0$$

$n+5=0$ or $n-6=0$

$n = -5$ (extraneous) or $n = 6$

(B) ${}_{n-1}P_2 = 12$



$$\frac{(n-1)!}{[(n-1)-2]!}$$

$$\frac{(n-1)!}{(n-3)!} = 12$$

$$\frac{(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 12$$

$$(n-1)(n-2) = 12$$

$$n^2 - 2n - n + 2 = 12$$

$$n^2 - 3n + 2 - 12 = 0$$

$$n^2 - 3n - 10 = 0$$

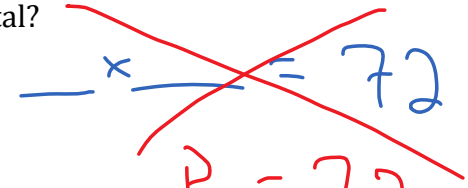
10
1,10
2,5

$$(n+2)(n-5) = 0$$

$n + 2 = 0$ $\quad$ $n - 5 = 0$

$n = -2$ $\quad$ $n = 5$

Example 2:
Lorna has a set of posters to arrange on her bedroom wall. She can only fit two posters side by side. If there are 72 ways to choose and arrange two posters, how many posters does she have in total?



$_nP_r = \frac{n!}{(n-r)!}$

$n = ?$
$r = 2$

$_nP_2 = 72$

$\frac{n!}{(n-2)!} = 72$

$\frac{n(n-1)(n-2)!}{(n-2)!} = 72$

$n^2 - n = 72$

$n^2 - n - 72 = 0$

$(n+8)(n-9) = 0$

$n = -8$, $n = 9$

Lorna has 9 posters.

Example 3:
Solve for n and state restrictions.

$_nP_r = \frac{n!}{(n-r)!}$

(A) $_nP_2 = 42$

$$\frac{n!}{(n-2)!} = 42$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 42$$

$$n^2 - n = 42$$

$$n^2 - n - 42 = 0$$

$$(n+6)(n-7) = 0$$

$n \neq -6$, $\boxed{n = 7}$

(B) $_{n-1}P_2 = 20$

$$\frac{(n-1)!}{[(n-1)-2]!} = 20$$

$$\frac{(n-1)!}{(n-3)!} = 20$$

$$\frac{(n-1)(n-2)(n-3)!}{(n-3)!} = 20$$

$$n^2 - 2n - n + 2 = 20$$

$$n^2 - 3n + 2 - 20 = 0$$

$$n^2 - 3n - 18 = 0$$

$$(n+3)(n-6) = 0$$

$n \neq -3$, $\boxed{n = 6}$

Textbook Questions: page 94; #15