Math 3201

2.6B Solving Combination Equations for n

These are done similar to solving permutation equations for n. $\binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!}$

Example 1:
Solve for n and state restrictions.

(A) ${}_nC_2 = 6$

$$\frac{n!}{2!(n-2)!} = 6$$

$$\cancel{2!}\cdot\frac{n(n-1)\cancel{(n-2)!}}{\cancel{2!}\,\cancel{(n-2)!}} = 6 \cdot 2!$$

$$n(n-1) = 12$$
$$n^2 - n - 12 = 0$$
$$(n+3)(n-4) = 0$$
$$n+3=0, \quad n-4=0$$
$$\cancel{n=-3}, \quad \boxed{n=4}$$

(B) ${}_nC_2 = 15$

$$\frac{n!}{2!(n-2)!} = 15$$

$$\cancel{2!}\cdot\frac{n(n-1)\cancel{(n-2)!}}{\cancel{2!}\,\cancel{(n-2)!}} = 15\cdot 2!$$

$$n(n-1) = 30$$
$$n^2 - n - 30 = 0$$
$$(n+5)(n-6) = 0$$
$$n+5=0, \quad n-6=0$$
$$\cancel{n=-5}, \quad \boxed{n=6}$$

(C) $\binom{n}{2} = 10$

$$\frac{n!}{2!(n-2)!} = 10$$

$$\cancel{2!}\cdot\frac{n(n-1)\cancel{(n-2)!}}{\cancel{2!}\,\cancel{(n-2)!}} = 10\cdot 2!$$

$$n(n-1) = 20$$
$$n^2 - n - 20 = 0$$
$$(n+4)(n-5)$$
$$n+4=0, \quad n-5=0$$
$$\cancel{n=-4}, \quad \boxed{n=5}$$

(D) $\dfrac{\cancel{2}\ {}_{n}C_{2}}{\cancel{2}} = \dfrac{30}{2}$

$_nC_2 = 15$

See (B) for entire solution.

(E) $_{n+1}C_1 = 20$

$$\frac{(n+1)!}{1!(n+1-1)!} = 20$$

$$\frac{(n+1)!}{1!n!} = 20$$

$$\frac{(n+1)\cancel{n!}}{\cancel{n!}} = 20 \rightarrow n+1 = 20$$

$n = 20 - 1$

$n = 19$

Textbook Questions: page 119; #15a