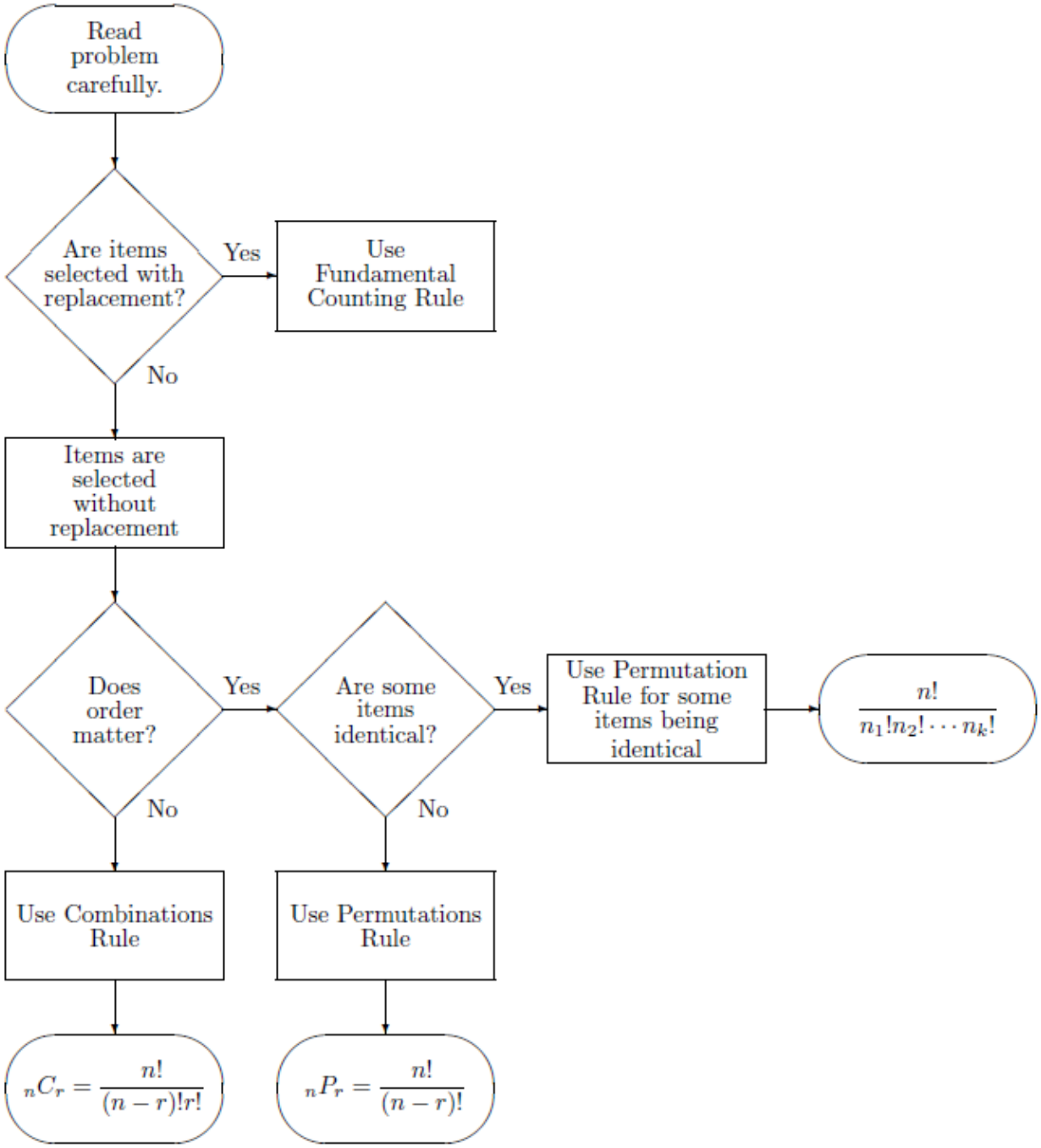
Math 3201

2.7 Solving Counting Problems

On this handout, we will look at a variety or problems in which we will have determine which method(s) we should use in solving. To assist with this decision making process, refer to the "Permutations and Combinations Flowchart".



Example 1:
A piano teacher and her students are having a group photograph taken. There are three boys and five girls. The photographer wants the boys to sit together and the girls to sit together for one of the poses. How many ways can the students and teacher sit in a row of nine chairs for this pose?

$$P = \underbrace{3 \times 2 \times 1}_{B\ B\ B} \times \underbrace{5 \times 4 \times 3 \times 2 \times 1}_{G\ G\ G\ G\ G} \times \frac{1}{T}$$

$$P = 3! \cdot 5!$$

$$P = 4320$$

Example 2:
Combination/Permutation problems are common in computer science. Suppose there is a set of 10 different data items represented by {a, b, c, d, e, f, g, h, i, j} to be placed into four memory cells in a computer. Only 3 data item are to be placed in the first cell, 4 data items in the second cell, 2 data items in the third cell, and 1 data item in the last cell. How many ways can the 10 data items be placed in the four memory cells?

3 out of 10 go in cell #1.
4 out of 7 go in cell #2.
2 out of 3 go in cell #3.
1 out of 1 go in cell #4.

$$_{10}C_3 \times {}_7C_4 \times {}_3C_2 \times {}_1C_1$$

$$= 12600$$

Example 3:
How many different five-card hands that contain at most one black card be dealt to one person from a standard deck of playing cards?

at most 1: 0 or 1 26 black, 26 red

0 black and 5 red: $_{26}C_0 \times {}_{26}C_5 = 1 \times 65780 = 65780$

or

1 black and 4 red: $_{26}C_1 \times {}_{26}C_4 = 26 \times 14950 = 388700$

+

$C_T = 454480$

Textbook Questions: page 126 - 127 #1 - 12, 14, 16