Math 3201

3.3 Probabilities Using Counting Methods

Review of Permutations and Combinations:

Permutations: used when the order of the arrangement is important
Combinations: used when the order of the arrangement is NOT important.

Recall that probability is calculated as follows:

$$P = \frac{favourable}{total}$$

In this section, we will use permutations and combinations to determine the number of favorable and the total number of outcomes.

Example 1:
A survey was conducted of 500 adults who wore Halloween costumes to a party. Each person was asked how he/she acquired the costume: created it, rented it, bought it, or borrowed it. The results are as follows: 360 adults created their costumes, 60 adults rented their costumes, 60 adults bought their costumes and 20 adults borrowed their costumes. What is the probability that the first four people who were polled all created their costumes.

(A) What counting strategy will you use? Is order important?

Combination. order not important.

(B) What are the total number of possible outcomes?

$_{500}C_4 = 2,573,031,125$

(C) What are the number of favourable outcomes? In other words what are the number of ways to choose 4 people from the group who created their own costumes?

$_{360}C_4 = 688\ 235\ 310$

(D) How would you use this information to determine the probability?

$$P = \frac{f}{t} = \frac{688\ 235\ 310}{2\ 573\ 031\ 125} = 0.267 \text{ or } 26.7\%$$

Example 2:
If a 4 digit number is generated at random from the digits 2, 3, 5 and 7 (without repetition of the digits), what is the probability that it will be even?

(A) What counting strategy will I use in this problem?

Permutation

(B) What information do I need to determine the probability?

$\text{Probability} = \frac{\text{fav's}}{\text{total}}$

(C) How will I determine the total number of 4 digit numbers that can be created from these 4 digits?

$T = 4 \times 3 \times 2 \times 1 = 4! = 24$

(D) What condition must be met in order for the number to be even?

$F = \_ \times \_ \times \_ \times \frac{1}{2}$

(E) How will I determine the total number of 4 digit even numbers?

$F = 3 \times 2 \times 1 \times \frac{1}{2} = 6$

(F) What is the probability that the number will be even?

$P = \frac{f}{t} = \frac{6}{24} = 0.25$ or 25% or 1:4

Example 3:
A 4 digit PIN number can begin with any digit, except zero, and the remaining digits have no restriction. If repeated digits are allowed, find the probability of the PIN code beginning with a number greater than 7 and ending with a 3.

$t = \frac{9}{\neq 0} \times 10 \times 10 \times 10 = 9000$

$f = \frac{2}{8 \text{ or } 9} \times 10 \times 10 \times \frac{1}{3} = 200$

$P = \frac{f}{t} = \frac{200}{9000}$

$= 0.022$

or

1:45 2.2%

Example 4:

Mark, Abby and 5 other students are standing in a line. Determine the probability Mark and Abby are standing together. Determine the probability Mark and Abby are not standing together.

$t = 7! = 5040$ (6)

$f = \underbrace{2 \times 1}_{M\ A} \times 5 \times 4 \times 3 \times 2 \times 1$

$= 2! \cdot 6! = 1440$

$P = \frac{f}{t} = \frac{1440}{5040}$

$= 0.286$

or 28.6%

$P(Am)' = 100\% - 28.6\% = 71.4\%$

Example 5:

A bookcase contains 6 different math books and 12 different biology books. If a student randomly selects two of these books, determine the probability they are both math or both biology books.

$t = {}_{18}C_2 = 153$

2 math or 2 biology

${}_6C_2 \times {}_{12}C_0 = 15 \times 1 = 15$

or

${}_{12}C_2 \times {}_6C_0 = 66 \times 1 = 66$

fav's: 81

$P = \frac{f}{t} = \frac{81}{153} = 0.53$ or 53%

Example 6:

A jar contains 5 orange, 7 blue, 3 purple and 5 green candies. If the total number candies is 20, determine the probability that a handful of four candies contains one of each colour.

$t = {}_{20}C_4 = 4845$

$f = {}_5C_1 \times {}_7C_1 \times {}_3C_1 \times {}_5C_1$

$= 5 \times 7 \times 3 \times 5$

$= 525$

$P = \frac{f}{t} = \frac{525}{4845} = 0.108$

or

10.8%

Example 7:
Two students passed in their response to the following question. There are 7 teachers and 3 administrators at a conference. Find the probability of three different door prizes being awarded to teachers only.

Student A $\dfrac{{}_7C_3 \times {}_3C_0}{{}_{10}C_3} = \dfrac{35}{120}$

Student B $\dfrac{{}_{10}C_3 - ({}_7C_2 \times {}_3C_1) - ({}_7C_1 \times {}_3C_2) - ({}_7C_0 \times {}_3C_3)}{{}_{10}C_3} = \dfrac{35}{120}$

(A) What does student's A equation represent?

(B) What does student's B equation represent?

(C) Which method would you prefer and why?

Textbook Questions: page 159 - 161: 2, 3, 5, 7, 8, 10, 11, 12, 13, 14