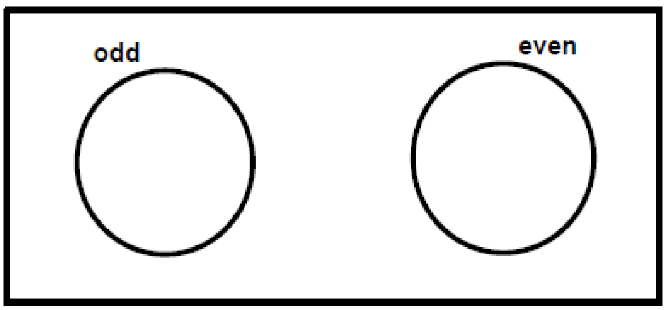
Math 3201

3.4 Mutually Exclusive vs. Non-Mutually Exclusive Events

Mutually Exclusive Events (Disjoint Sets): sets that have no common elements. That is, they never intersect.

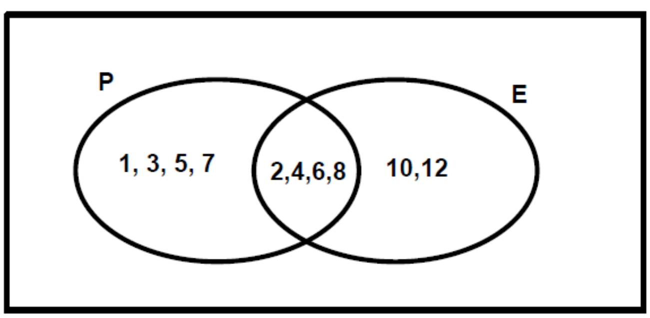
Example: the set of odd integers and the set of even integers would be mutually exclusive since they have no common elements.



Notice that there is no overlaping region between the sets on the Venn Diagram.

Non-Mutually Exclusive Events (Overlapping Sets): these are sets that share common elements. They intersect on a Venn Diagram.

Example: the set of positive integers from 1 to 8 and the even numbers from 1 to 12.



Notice that 2, 4, 6, 8 are in the overlaping region between the two sets. This means that the sets intersect and that the elements 2, 4, 6, 8 are common to both.

Example 1:
Classify the events in each experiment as being either mutually exclusive or non-mutually exclusive.

(A) The experiment is rolling a die. The first event is rolling an even number and the second event is rolling a prime number.

roll di: 1, 2, 3, 4, 5 6   prime: 1, 2, 3, 5   non-mutually exclusive.

(B) The experiment is playing a game of hockey. The first event is that your team scores a goal, and the second event is that your team wins the game.

You have to score to win. ∴ non-mutually exclusive

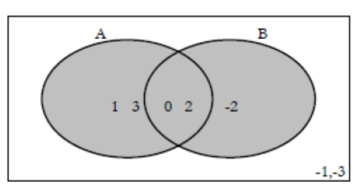
(C) The experiment is selecting a gift. The first event is that the gift is edible and the second event is that the gift is an iPhone.

no overlap. mutually exclusive

Review of Venn Diagrams Terminology

Union of Sets (OR)

- This represents all of the elements that are in one set OR the other set OR in the overlap between the sets.
- it is often indicated using the word "or"
- the symbol for union is ∪

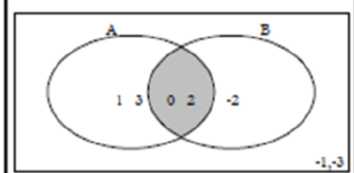
Set Notation	Meaning	Venn Diagram	Answer
A ∪ B (A union B)	any element that is in either of the sets		{-2, 0, 1, 2, 3}

Intersection of Sets (AND)

· This represents all of the elements that are in the overlap between the sets.

· it is often indicated using the word "and"

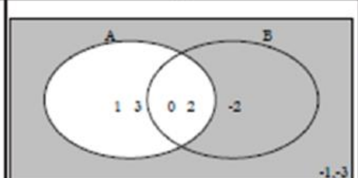
· the symbol for intersection is ∩

Set Notation	Meaning	Venn Diagram	Answer
A∩B (A intersect B)	only elements that are in both A and B		{0,2}

Complement of a Set

· This includes any element that is NOT in the set.

· The complement of set A can be written in two ways: A' or not A

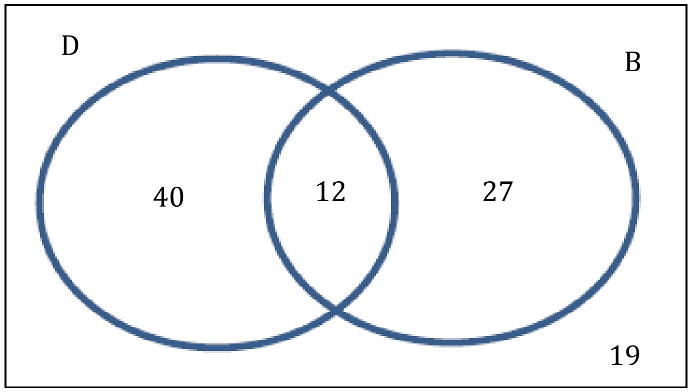
Set Notation	Meaning	Venn Diagram	Answer
A' (A complement or not A)	all elements in the universal set outside of A		{-3, -2, -1}

NOTE: the number of elements in set A, and the number of elements in the complement of Set A make up the number of elements in the entire set.

That is: $n(\mathrm{A}) + n(\mathrm{A}') = \text{total}$

Review: Calculating the Number of Elements in a Union of Sets

Consider the following example in which D represents students on the debate team and B represents students on the basketball team:



QUESTIONS:

1. Are the sets intersecting or disjoint?

2. How many students are on the debate team?

   $40 + 12 = 52$

3. How many students are on the basketball team?

   $27 + 12 = 39$

4. How many students are in the universal set (total)?

   $40 + 12 + 27 + 19 = 98$

5. Write a formula that we could use to determine the number of students in the union of sets D and B.

$$n(D \cup B) = n(D) + n(B) - n(D \cap B)$$
$$= 52 + 39 - 12$$
$$= 79$$

or $40 + 12 + 27 = 79$

Calculating the Probability of Non-Mutually Exclusive Events

Recall the formula that we just came up with:

$$n(\mathrm{D}\cup\mathrm{B}) = n(\mathrm{D}) + n(\mathrm{B}) - n(\mathrm{D}\cap\mathrm{B})$$

Right now, this represents the number of elements in the union of sets D and B. We can use this to determine the probability of an event being found in the overlap between the two sets. All we need to do ~~it~~ is to divide the three terms, which would be considered favorable outcomes, by the total number of outcomes, which we will call $n(\mathrm{S})$.

$$n(\mathrm{D}\cup\mathrm{B}) = n(\mathrm{D}) + n(\mathrm{B}) - n(\mathrm{D}\cap\mathrm{B})$$

$$P(\mathrm{D}\cup\mathrm{B}) = \frac{n(\mathrm{D}) + n(\mathrm{B}) - n(\mathrm{D}\cap\mathrm{B})}{n(\mathrm{S})}$$

intuitively use this.

$$P(\mathrm{D}\cup\mathrm{B}) = \frac{n(\mathrm{D})}{n(\mathrm{S})} + \frac{n(\mathrm{B})}{n(\mathrm{S})} + \frac{n(\mathrm{D}\cap\mathrm{B})}{n(\mathrm{S})}$$

$$P(\mathrm{D}\cup\mathrm{B}) = P(\mathrm{D}) + P(\mathrm{B}) - P(\mathrm{D}\cap\mathrm{B})$$ ← Given

Probability Formula for Non-Mutually Exclusive Events

In general, if sets A and B overlap, then the probability of either A or B occurring would be given by the formula:

$$P(A\cup B) = P(\mathrm{A}) + P(\mathrm{B}) - P(\mathrm{A}\cap\mathrm{B}) \longleftarrow \text{provided for assessments}$$

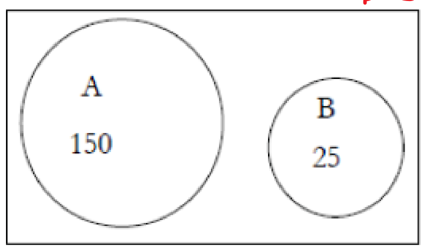
Probability Formula for Mutually Exclusive Events

Mutually exclusive events don't have any overlap. Thus, $n(A\cap B) = 0$. If we substitute this into the probability formula, we get:

$$P(A\cup B) = P(\mathrm{A}) + P(\mathrm{B}) \longleftarrow \textbf{NOT} \text{ provided for assessments}$$

Example 1:
Determine the $P(A \cup B)$ using the Venn Diagram below. The sample space has 500 outcomes.

500



Formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{150}{500} + \frac{25}{500} - 0$$
$$= \frac{175}{500}$$
$$= 0.35 \text{ or } 35\%$$

or

$$P = \frac{f}{t} = \frac{150+25}{500}$$
$$= \frac{175}{500}$$
$$= 0.35 \text{ or } 35\%$$

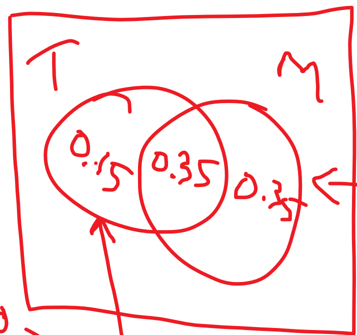
Example 2:
The probability that Dana will make the hockey team is $\frac{2}{3}$. The probability that she will make the swimming team is $\frac{3}{4}$. If the probability of Dana making both teams is $\frac{1}{2}$, determine the probability that she will make at least one of the teams.

intersect $\cap$
$P(H \cap S)$

union $\cup$
$P(H \cup S)$

$$P(H \cup S) = P(H) + P(S) - P(H \cap S)$$
$$= \frac{2}{3} + \frac{3}{4} - \frac{1}{2}$$
$$= 0.667 + 0.75 - 0.5$$
$$= 0.92 \text{ or } 92\%$$

Example 3:
The probability that the Toronto Maple Leafs will win their next game is 0.5. The probability that the Montreal Canadiens will win their next game is 0.7. The probability that they will both win is 0.35. Create a Venn Diagram of the situation and determine the probability that one or the other will win. Assume they don't play each other.



$0.7 - 0.35$
$= 0.35$

$0.5 - 0.35 = 0.15$

$P(T \cup M) = 0.15 + 0.35 + 0.35$
$= 0.85$ or 85%

Example 4:
A school newspaper published the results of a recent survey. It showed the following:

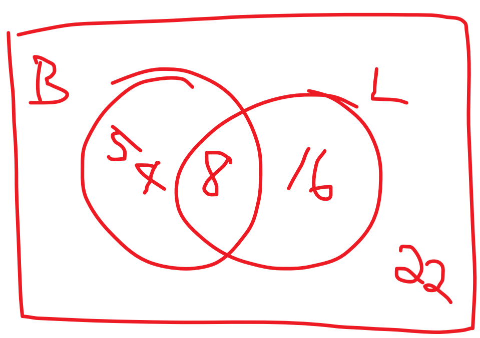
62% skip breakfast
24% skip lunch
22% eat both breakfast and lunch
108%

If total % exeeds 100, we have overlap. ∴ non-mutually exclusive.

(A) Are skipping breakfast and skipping lunch mutually exclusive?

No. The extra 8% represent people in the overlap. ∴ non-mutually exclusive.

(B) Create a Venn Diagram of the situation.



(C) Determine the probability that a randomly selected student skips breakfast but not lunch.

$P(B/L) = 54\%$

(D) Determine the probability that a randomly selected student skips at least one of breakfast or lunch.

$P(B \cup L) = 54 + 8 + 16 = 78\%$

or

$P(B \cup L) = 62 + 24 - 8 = 78\%$

Example 5

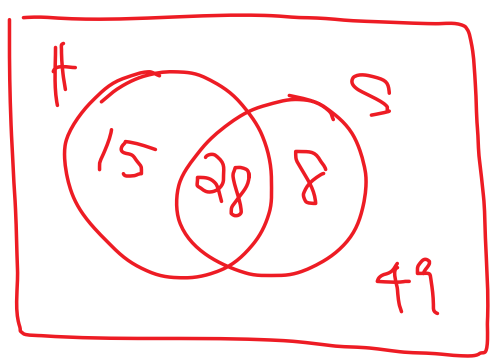
A car manufacturer keeps a database of all the cars that are available for sale at all dealerships in Western Canada. For model A, the database reports that 43% have heated leather seats, 36% have a sunroof, and 49% have neither.

There is 28% overlap.

(A) Are the events mutually exclusive?

$43 + 36 + 49 = 128\%$ $\therefore$ non-mutually exclusive.

(B) Create a Venn Diagram of the situation.



$\cap$

(C) Determine the probability of car model A having BOTH heated leather seats and a sunroof.

$P(H \cap S) = 28\%$

Probabilities of Complementary Events

Example 6:
Suppose you are rolling a fair 6 sided di.

(A) What is the probability of rolling a 2?

$P(2) = \frac{1}{6} = 0.17$ or 17%

(B) What is the probability of NOT rolling a 2?

$P(2)' = \frac{5}{6} = 0.83$ or 83%

(C) What do you notice about the two probabilities?

$P(2) + P(2)' = 100\%$

$\frac{1}{6} + \frac{5}{6} = \frac{6}{6} = 1$ or 100%

Complementary events have probabilities that add up to give 1.

$$P(\mathrm{A}) + P(\mathrm{A}') = 1$$

Rearranging, we can find the probability of the complement of event A using.

$$P(\mathrm{A}') = 1 - P(\mathrm{A})$$

Recall **Example 2:**
The probability that Dana will make the hockey team is $\frac{2}{3}$. The probability that she will make the swimming team is $\frac{3}{4}$. If the probability of Dana making both teams is $\frac{1}{2}$, determine the probability that she will make at least one of the teams.

Now calculate the probability that she will make neither team.

$P(H \cup S) = 0.92$ or 92%

$P(H \cup S)' = 1 - 0.92 = 0.08$

or

$= 100\% - 92\% = 8\%$

Example 7:

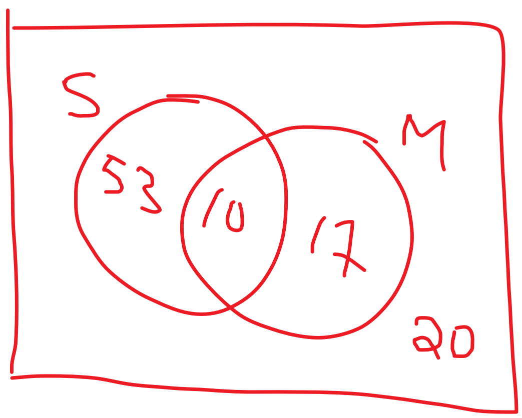
Class Survey
63% of students play sports
27% of students play a musical instrument
20% of students play neither sports nor a musical instrument.

(A) Are the events mutually exclusive?

10% overlap

$63\% + 27\% + 20\% = 110\% \therefore$ non-mutually exclusive

(B) Create a Venn Diagram.



$\cup$

(C) Determine the probability that a student will either play sports or play a musical instrument.

$P(S \cup M) = 53\% + 10\% + 17\% = 80\%$

Textbook Questions: pages 176 – 179: 3, 4, 5, 6, 7, 8, 13, 14, 15