Math 3201

3.5 Conditional Probability

Dependent Events: Events whose outcomes are affected by each other. For example, if two cards are drawn from a deck without replacement, the outcome of the second event depends on the outcome of the first event which is the first card drawn.

Conditional Probability: The probability of an event occuring given that another event has already occurred.

The methods that we will use to solve probability problems involving dependent events are very similar to what we used on Lesson 5 when dealing with independent events. The only difference that we will see is in the way that the probability for the second event is calculated. Since the second events depends on the first, the favorable and total outcomes for the second event will need to be adjusted accordingly.

Formula for Conditional Probability
The formula for conditional probability is very similar to that for the probability of two independent events, with the exception that we must take into account that the probability of the second event depends upon the probability of the first event.

Review:

Probability Formula for Two Independent Events A and B Both Occurring

$$P(\mathrm{A} \cap \mathrm{B}) = P(\mathrm{A}) \times P(\mathrm{B})$$

Probability for Dependent Events or Conditional Probability
Since B depends on A, we use the notation $P(\mathrm{B}|\mathrm{A})$. This means the probability of B given that A has already occurred. Thus the probability of both A and B occurring can be represented by:

$$P(A \cap B) = P(A) \times P(\mathrm{B}|\mathrm{A})$$

Example 1:
Suppose we have a standard deck of 52 cards. What is the probability of selecting two diamonds:

(A) With replacement.

$P(A) = \frac{13}{52} = \frac{1}{4}$ $P(B) = \frac{13}{52} = \frac{1}{4}$

A: 1st diamond

B: 2nd diamond

$P(A \cap B) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} = 0.0625$

or 6.25%

(B) Without replacement.

$$P(A) = \frac{13}{52} = \frac{1}{4} \qquad P(B|A) = \frac{12}{51}$$

$$P(A \cap B) = \frac{1}{4} \times \frac{12}{51} = \frac{12}{204} = \frac{1}{17} = 0.0588$$

or

5.88%

Example 2:

Cards are drawn from a standard deck of 52 cards without replacement. Calculate the probability of obtaining:

(A) a king, then another king

A B

$$P(A) = \frac{4}{52} \qquad P(B|A) = \frac{3}{51}$$

$$P(A \cap B) = \frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} = \frac{1}{221} = 0.00452$$

or 0.452%

(B) a club, then a heart

A B

$$P(A) = \frac{13}{52} \qquad P(B|A) = \frac{13}{51}$$

$$P(A \cap B) = \frac{13}{52} \times \frac{13}{51} = \frac{169}{2652} = \frac{13}{204} = 0.0637$$

or 6.37%

(C) a black card, then a heart, then a diamond

A B C

$$P(A) = \frac{26}{52} = \frac{1}{2}, \quad P(B|A) = \frac{13}{51}, \quad P[C|(B|A)] = \frac{13}{50}$$

$$P(A \cap B \cap C) = \frac{1}{2} \times \frac{13}{51} \times \frac{13}{50} = \frac{169}{5100} = 0.0331 \text{ or } 3.31\%$$

Example 3:
A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

$P(B \cap W) = 0.34$

$P(B) = 0.47$

$P(W|B) = ?$

$P(B \cap W) = P(B) \times P(W|B)$

$\frac{0.34}{0.47} = \frac{\cancel{0.47} \cdot P(W|B)}{\cancel{0.47}}$

$P(W|B) = 0.72$ or 72%

Example 4:
A hockey team has jerseys in three different colors. There are 4 green, 6 white and 5 orange jerseys in the hockey bag. Todd and Blake are given a jersey at random. Students were asked to write an expression representing the probability that both jerseys are the same color. Which student correctly identified the probability and why?

3 cases: green or white or orange

Case 1: 2 green $\frac{4}{15} \times \frac{3}{14}$

+

Case 2: 2 white $\frac{6}{15} \times \frac{5}{14}$

+

Case 3: 2 orange $\frac{5}{15} \times \frac{4}{14}$

~~Tony~~	~~$\left(\frac{2}{4}\right)\left(\frac{2}{6}\right)\left(\frac{2}{5}\right)$~~
Sam	$\left(\frac{2}{4}\right)+\left(\frac{2}{6}\right)+\left(\frac{2}{5}\right)$
Leslie	$\left(\frac{4}{15}\right)\left(\frac{3}{14}\right)+\left(\frac{6}{15}\right)\left(\frac{5}{14}\right)+\left(\frac{5}{15}\right)\left(\frac{4}{14}\right)$
Dana	$\left(\frac{4}{15}\right)\left(\frac{4}{15}\right)+\left(\frac{6}{15}\right)\left(\frac{6}{15}\right)+\left(\frac{5}{15}\right)\left(\frac{5}{15}\right)$

Example 5:
A computer manufacturer knows that, in a box of 100 computer chips, 3 will be defective. Alicia will draw 2 chips, at random, from a box of 100 chips. Determine the probability that she will draw two defective chips.

A: 1st chip
B: 2nd chip

$P(A) = \frac{3}{100}$ $P(B|A) = \frac{2}{99}$

$P(A \cap B) = \frac{3}{100} \times \frac{2}{99} = \frac{6}{9900} = \frac{1}{1650}$

$= 0.000606$
$= 0.06\%$

Example 6:
In the Canadian Senate, there are approximately 100 senators. 63 of whom are Conservative and 37 Liberal. In a recent vote, the results were as follows:

	Conservative	Liberal	Total
Yes	32	20	52
No	31	17	48
Total	63	37	100

(A) When a senator is chosen at random, what is the probability that he/she is Conservative?

$\frac{63}{100}$ or 63%

(B) When a senator is chosen at random, what is the probability that he/she is Liberal?

$\frac{37}{100}$ or 37%

(C) When a senator is chosen at random, what is the probability that he/she:

(i) voted yes? $\frac{52}{100}$ or 52%

(ii) voted no? $\frac{48}{100}$ or 48%

(D) What is the probability that he/she voted yes **and** is Conservative?

$$\frac{52}{100} \times \frac{63}{100} = \frac{3276}{10000} = 0.327 \text{ or } 32.7\%$$

Now let's take a look at conditional probability.

(E) What is the probability that the senator voted yes **knowing** he/she is Conservative?

$$\frac{32}{63} = 0.510 \text{ or } 51\%$$

Textbook Questions: page 188 - 191; #1, 4, 7, 9, 18, 19