Math 3201

3.6 Independent Events

To determine whether two events are dependent or independent, determine whether one event will affect the probable outcome of another. If this doesn't happen, then the two events are said to be **independent** of each other. If one event does affect the other, then they are said to be **dependent**, and we use something called conditional probability.

For each example, determine whether the following sets of events are dependent or independent.

Example 1:

Event A: drawing a queen from a standard deck of cards
Event B: drawing a king from the remaining cards in the same deck

Dependent

Example 2:

Event A: rolling a 5 on a di
Event B: rolling a 3 on the same di

Independent

Example 3:
Classify the following events as either independent or dependent and explain why.

(A) The experiment is rolling a di and flipping a coin. The first is rolling a six and the second event is obtaining tails.

Independent. Di and coin have no effect on each other.

(B) The experiment is rolling a pair of dice. The first event is rolling an odd number on one di and the second event is rolling an even number on the other di.

Independent. First di has no effect on the second di.

(C) The experiment is dealing 5 cards from a standard deck. The first event is that the first card dealt is a spade, the second event is that the second card is a spade, the third event is that the third card is a spade and so on.

Dependent. 2nd event depends on the first and the 3rd event depends on on the 1st and second event, and so on. (#spades and #cards change)

(D) The experiment is to sample two members of a family, a mother and her child. The first event is that the mother has blond hair and the second event is that the child has blond hair.

Dependent. Hair colour is genetic. (complicated)

Review of Key Concepts

"Or" is represented by ∪ or union, and it means we **add**.

"**And**" is represented by ∩ or intersection, and it means we **multiply**.

When two events A and B are independent of each other, we can find the probability of A and B happening by using:

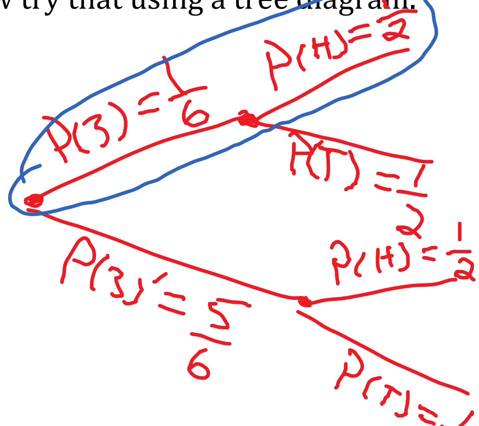
$$P(A \cap B) = P(A) \times P(B)$$

It is convenient to determine the probability of two independent events using a tree diagram.

Example 4:
Determine the probability of rolling a 3 on a di and tossing heads on a coin. Do **not** use a tree diagram.

$$P(3 \cap H) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12} = 0.083 \text{ or } 8.3\%$$

Now try that using a tree diagram.



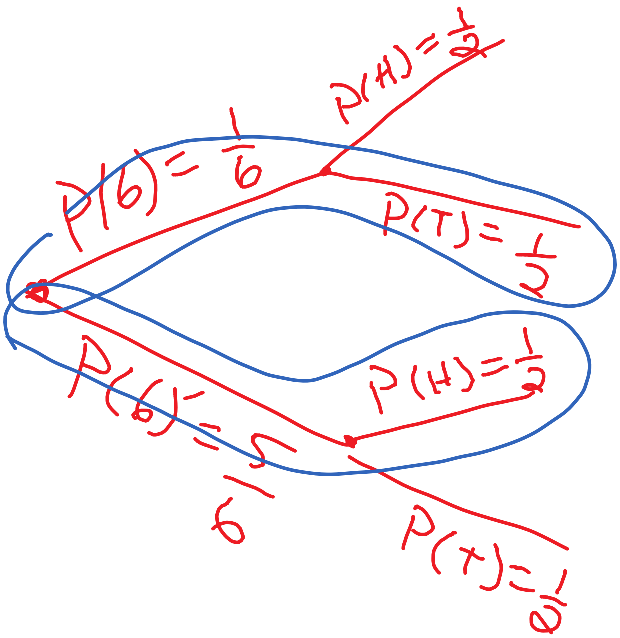
$$P(3 \cap H) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12} = 0.083 \text{ or } 8.3\%$$

What other information can you determine using the tree diagram?

$$P(3 \cap T) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{2}, \quad P(3' \cap H) = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}, \quad P(3' \cap T) = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$$

Example 5:
Vanessa and Erica are playing a dice and coin game. Each turn consists of rolling a regular di and tossing a coin. The goal of the game is to roll a 6 on the di and toss a head on the coin.

(A) Draw a tree diagram showing the probabilities of the various events.



(B) What is the probability of getting either a 6 OR a head, but not both?

$P(6 \cap H')$ or $P(6' \cap H)$

$= \frac{1}{6} \times \frac{1}{2} + \frac{5}{6} \times \frac{1}{2} = \frac{1}{12} + \frac{5}{12} = \frac{6}{12} = \frac{1}{2}$

(C) What is the probability of getting a 6 and a head?

$P(6 \cap H) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

(D) What is the probability of getting neither a 6 nor a head?

$P(6' \cap H') = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$

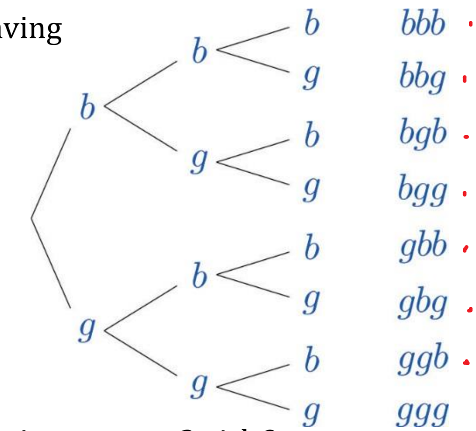
Example 6:

The tree diagram shown represents the possibilities for a couple having three children. total = 8

(A) What are the odds in favour of the couple having 2 girls?

f = 3   3:5

u = 8 - 3 = 5



(B) What are the odds in favour of the couple having at most 2 girls?

at most 2: 0 or 1 or 2

favs: 7   7:1

unfavs: 8 - 7 = 1

(C) What are the odds in favour of the couple having at most 1 boy?

at most 1: 0 or 1

favs = 4   4:4 or 1:1

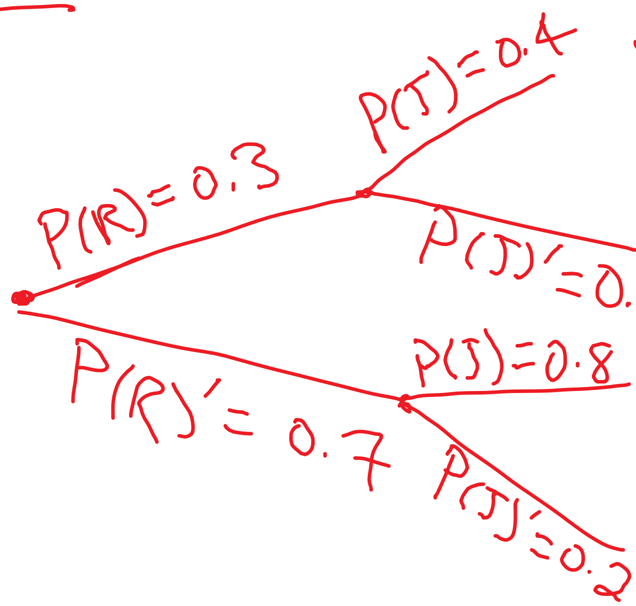
unfavs = 8 - 4 = 4

(D) What is the probability of the couple having at least 1 boy?

at least 1: 1 or 2 or 3

favs: 7 $P = \frac{7}{8} = 0.875$ or 87.5%

Example 7:
John likes to jog. If the weather is nice he is 80% likely to jog. If it is raining he is only 40% likely to jog. The forecast for tomorrow indicates a 30% chance of rain. What is the percent probability that he will jog tomorrow? Change to decimals!



$P(R) \times P(J|R) = 0.3 \times 0.4 = 0.12$

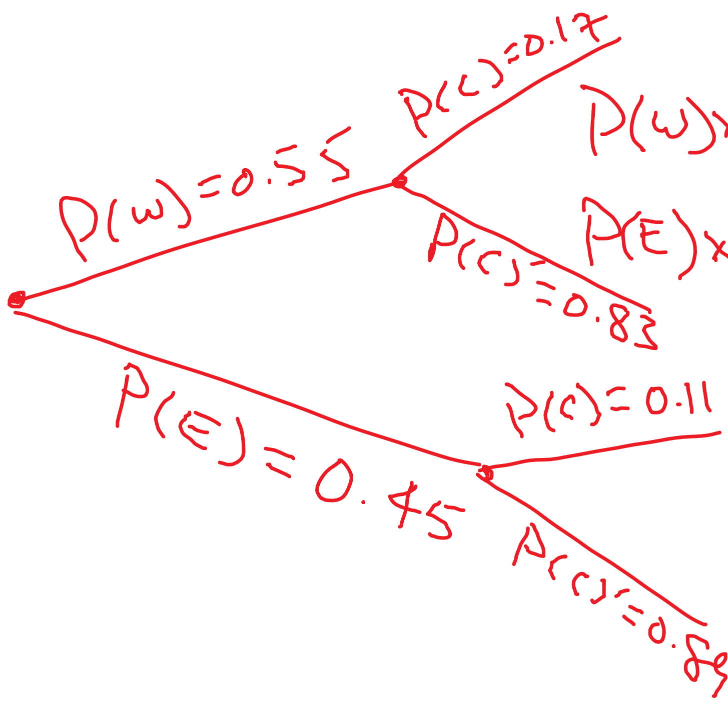
or +

$P(R') \times P(J|R') = 0.7 \times 0.8 = 0.56$

$\overline{0.68}$

There is a 68% chance John will jog.

Example 8:
In a junior football league, 55% of the players come from Western Canada and 45% come from Eastern Canada. From this league, 17% of the Western Players and 11% of the Eastern Players go on to the CFL. If a randomly chosen CFL player who came from this junior league is selected, what is the probability that he came from Eastern Canada, P(Eastern Canada|CFL), to the nearest tenth of a percent?



$P(W) \times P(C|W) = 0.55 \times 0.17 = 0.0935$

$P(E) \times P(C|E) = 0.45 \times 0.11 = 0.0495$

total $= 0.0935 + 0.0495$
$= 0.143$

$P(E|C) = \dfrac{0.0495}{0.143}$
$= 0.346$ or 34.6%

Textbook Questions: page 198, 199; #1, 5, 6, 7, 8, 9