Math 3201
4.1A Rational Expressions and Non-Permissible Values

A rational expression is any expression that can be written as the quotient of two polynomials, in the form $\frac{P(x)}{Q(x)}$ where $Q(x) \neq 0$.

A rational expression MUST have at least one variable in both the numerator and denominator. Otherwise, it is just an algebraic fraction.

Question: Why can't the denominator of the fraction equal 0?

Can't divide by 0.

Example 1:
Classify the following as being either rational expressions or algebraic fractions.

$$\frac{4}{5}, \quad \frac{2x}{y}, \quad \frac{x^2-4}{x+1}, \quad \sqrt{5}, \quad 2\pi, \quad \frac{\sqrt{x}}{2y}, \quad \frac{x^2}{4}$$

not a variable

Non-Permissible Values of a Rational Expression
These are values of a variable that make the denominator equal zero. That is, when we substitute these values into the denominator, it becomes zero.

Example 2:
Explain why $x = 2$ is a non-permissible value for $\frac{3x}{x-2}$.

If we sub 2 in for x.

$$\frac{3(2)}{2-2}$$

∴ x ≠ 2

$$= \frac{6}{0} \leftarrow \text{undefined}$$

Steps for finding non-permissible values:
· Take the denominator and set it equal to zero.

· If the equation you come up with is linear, just solve for x.

· If the equation you come up with is quadratic, try just solving for x. If this is not possible, try factoring OR using the quadratic formula.

* What makes denominator 0?

Example 3:
Determine non-permissible values for each rational expression.

(A) $\dfrac{4x^2+8x}{4x}$

$$\dfrac{4x}{4} \neq \dfrac{0}{4}$$

$$x \neq 0$$

(E) $\dfrac{5x-2}{4x^2-16}$

$$= \dfrac{5x-2}{(2x+4)(2x-4)}$$

$$2x+4 \neq 0 \quad \to x \neq -2$$
$$\dfrac{2x}{2} \neq \dfrac{-4}{2}$$

$$2x-4 \neq 0$$
$$\dfrac{2x}{2} \neq \dfrac{4}{2}$$
$$x \neq 2$$

(B) $\dfrac{x}{x+2}$

$$x+2 \neq 0$$
$$x \neq -2$$

(F) $\dfrac{-15}{x^2-5x}$

$$x(x-5) \neq 0$$
$$x \neq 0 \quad , \quad x-5 \neq 0$$
$$x \neq 5$$

(C) $\dfrac{4x^3}{4-x}$

$$4-x \neq 0$$
$$4 \neq x$$
or
$$x \neq 4$$

(G) $\dfrac{-5x-2}{20x^2+15x}$

$$5x(4x+3) \neq 0$$
$$\dfrac{5x}{5} \neq \dfrac{0}{5} \qquad 4x+3 \neq 0$$
$$x \neq 0 \qquad \dfrac{4x}{4} \neq \dfrac{-3}{4}$$
$$x \neq -3/4$$

(D) $\dfrac{x-1}{3x^2-12}$

$$3(x^2-4)$$
$$3(x+2)(x-2)$$
$$x+2 \neq 0 \quad , \quad x-2 \neq 0$$
$$x \neq -2 \quad , \quad x \neq 2$$

or $3x^2-12 \neq 0$
$$\dfrac{3x^2}{3} \neq \dfrac{12}{3}$$
$$\sqrt{x^2} \neq \sqrt{4}$$
$$x \neq \pm 2$$

why -2?
Since $(-2)^2 = 4$
then $\sqrt{4} = -2$
-2 is secondary root.
2 is principle root.

Example 4:

Write a rational expression that has non-permissible values of:

(A)   2 and −4

$$X \neq 2 \quad , X \neq -4$$
$$(X-2) \neq 0 \quad , (X+4) \neq 0$$

$$\frac{X}{(X-2)(X+4)}$$

(B)   0, −2 and 3

$$(X) \neq 0 \quad , X \neq -2 \quad X \neq 3$$
$$(X+2) \neq 0 \quad {}'(X-3) \neq 0$$

$$\frac{X}{X(X+2)(X-3)}$$

(C) $\frac{1}{4}$ $X \neq \frac{1}{4}$ $\to (4X-1) \neq 0$

$$4X \neq 1$$

$$\frac{X}{4X-1}$$

Non-Permissible vs. Inadmissible Values for a Variable

Non-Permissible Values: values of a variable that make the denominator of a rational expression equal 0.

Inadmissible Values: values of a variable that do NOT make sense in the context of a given problem.

Example 5:

Suppose the expression $\frac{20}{x}$ is used to represent the time taken to complete a trip.
$x \neq 0$ is a non-permissible value since it makes the denominator equal zero negative x-values are inadmissible since they result in negative time values which doesn't make sense!

Example 6:

What are the non-permissible values for $\frac{x+3}{x^2-16}$?

① I think the non-permissible value is 4.

② I think the non-permissible values are -4 and 4.

③ I think the non-permissible values are -4, -3 and 4.
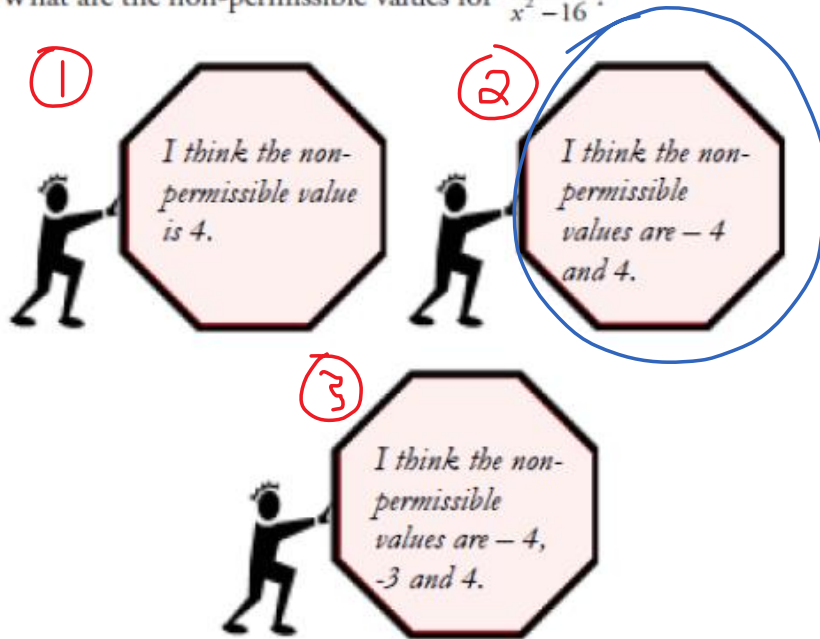
Who is correct? Justify your answer by solving the problem.

$$x^2 - 16 \neq 0$$
$$(x+4)(x-4) \neq 0$$
$$x+4 \neq 0, \quad x-4 \neq 0$$
$$x \neq -4 \qquad x \neq 4$$

Textbook Questions: page 223, 224 #3 find non-permissible values, 9 a), 10, 11 a), 16