Math 3201

4.1B Equivalent Rational Expressions

Review:

Writing Equivalent Fractions

We can write a fraction that is equivalent to a given fraction by multiplying both the numerator and denominator by the same number.

Example 1:
Write a fraction that is equivalent to $\frac{2}{3}$.

$$\frac{2 \times 2}{3 \times 2} = \frac{4}{6} \text{ is equivalent to } \frac{2}{3}.$$

When we multiply top and bottom of a fraction by the same value, we are essentially multiplying by 1. That is why the numerical value of the equivalent fractions remains the same, even though the two fractions may look different.

We can do something similar with rational expressions. Multiplying top and bottom of a rational expression by the same number will result in an equivalent rational expression. The same is true if we multiply top and bottom by some polynomials. However, if we multiply top and bottom by a polynomial that ends up changing the restrictions on our new rational expression, then the two expressions will NOT be equivalent.

Example 2:
Consider the rational expression $\frac{4}{x}$.

(A) Multiply top and bottom by 2. Did the restrictions change on the new rational expression? Is the new expression equivalent to the original one?

$$\frac{4 \cdot 2}{x \cdot 2} = \frac{8}{2x}$$

$x \neq 0$ $\quad$ $x \neq 0$

i. top and bottom multiplied by same value.

ii. Same non-permissible values.

(B) Multiply top and bottom by x. Did the restrictions change on the new rational expression? Is the new expression equivalent to the original one?

$$\frac{4}{x} \cdot \frac{x}{x} = \frac{4x}{x^2}$$

$x \neq 0$

$\sqrt{x^2} \neq \sqrt{0}$

$x \neq 0$

both i & ii are met.
∴ equivalent fractions

(C) Multiply top and bottom by $x - 1$. Did the restrictions change on the new rational expression? Is the new expression equivalent to the original one?

$$\frac{4}{x} \cdot \frac{(x-1)}{(x-1)} = \frac{4x-4}{x^2-x}$$

$x \neq 0$

$x^2 - x \neq 0$

$x(x-1) \neq 0$

$x \neq 0, x-1 \neq 0$

$x \neq 1$

Only criteria i is met.
∴ not equivalent fractions

Summary:
To determine whether two rational expressions are equivalent:
· The top and bottom of one expression must be multiplied by the same value to give the other expression.
· The two expressions must have the same restrictions.

Example 3:
Determine whether the following sets of rational expressions are equivalent.

(A) $\frac{9}{3x-1}$ and $\frac{-18}{2-6x}$

$$\frac{9}{3x-1}\cdot\frac{(-2)}{(-2)} = \frac{-18}{-6x+2} = \frac{-18}{2-6x} \checkmark$$

criteria i is met.

$$3x-1 \neq 0 \qquad -6x+2 \neq 0$$

$$3x \neq 1 \qquad -6x \neq -2$$

$$\frac{3x}{3} \neq \frac{1}{3} \qquad \frac{-6x}{-6} \neq \frac{-2}{-6}$$

$$x \neq 1/3 \qquad x \neq \frac{1}{3}$$

criteria ii is met. $\therefore$ equivalent expressions

(B) $\frac{2-2x}{4x}$ and $\frac{x-1}{2x}$

$$\frac{(x-1)\cdot 2}{2x\cdot 2} = \frac{2x-2}{4x} = \frac{-2+2x}{4x}$$

Criteria i is not met.

∴ non-equivalent expressions.

Example 4:

Your friend thinks the expressions $\frac{3}{2x}$ and $\frac{3(x+1)}{2x(x+1)}$ are equivalent. Explain why these expressions are not equivalent.

Expression one is multiplied by the same on top and bottom. Criteria i is met.

$$\frac{3}{2x} \qquad \frac{3(x+1)}{2x(x+1)}$$

$$\frac{2x}{2} \neq \frac{0}{2} \qquad x \neq 0,\ x+1 \neq 0$$

$$x \neq 0 \qquad x \neq -1$$

Different non-permissible values. Criteria ii is not met. ∴ non equivalent expressions.

Example 5:
Complete the following table.

Are the expressions equivalent?	Yes	No	Justify your choice
$\frac{x+3}{x-4}$ and $\frac{4x+12}{4x-16}$	✓		i. ✓ ii. ✓
$\frac{5}{x-5}$ and $\frac{5x+25}{x^2-25}$		✓	i. ✓ ii. X
$\frac{x+2}{x-3}$ and $\frac{3x+6}{2x-6}$		✓	i. X

Textbook Questions: page 222 - 224 #1, 4, 5, 9 (b parts i and ii only)