Math 3201

4.4 Adding and Subtracting Rational Expressions

Review of Adding/Subtracting Fractions

Steps:

- Reduce the fractions first if possible.
- Get a common denominator.
- Rewrite each fraction with equivalent fractions so that each one has the common denominator.
- Add/subtract the numerators.
- Keep the common denominator the same.
- Reduce/simplify the answer if possible.

Example 1:

Simplify the following.

(A) $\frac{6}{7}+\frac{2}{7}$

$= \frac{6+2}{7}$

$= \frac{8}{7}$

(B) $\frac{4}{5}-\frac{1}{5}$

$= \frac{4-1}{5}$

$= \frac{3}{5}$

(C) $\frac{1}{3}+\frac{7}{12}$

$= \frac{4\cdot 1}{4\cdot 3}+\frac{7}{12}$

$= \frac{4}{12}+\frac{7}{12}$

$= \frac{4+7}{12}$

$= \frac{11}{12}$

(D) $\frac{1}{3}+\frac{3}{4}$

$= \frac{4\cdot 1}{4\cdot 3}+\frac{3\cdot 3}{3\cdot 4}$

$= \frac{4}{12}+\frac{9}{12}$

$= \frac{4+9}{12}$

$= \frac{13}{12}$

Example 2
Complete the table:

Rational Number	LCM	Rational Expression	LCM	Similarities
① $\frac{4}{5}+\frac{3}{5}$	5	$\frac{6}{2x-1}+\frac{-2}{2x-1}$	$2x-1$	Denominators are common
② $\frac{1}{5}-\frac{7}{15}$	15	$\frac{4x}{x-3}-\frac{5}{6x-18}$	$6x-18$	2nd is the common denominator.
③ $\frac{7}{12}+\frac{3}{8}$	24	$\frac{2}{x^2-36}+\frac{4}{3x+18}$	$3(x+6)(x-6)$	Had to find the LCD.

$$\frac{2}{(x+6)(x-6)}+\frac{4}{3(x+6)}$$

Example 3:

$$\frac{x^2}{x+1} - \frac{1}{x-1}$$

LCD: $(x+1)(x-1)$

What you want both denominators to look like.

$a \cdot b = b \cdot a$

$$= \frac{(x-1)\cdot x^2}{(x-1)\cdot(x+1)} - \frac{(x+1)\cdot 1}{(x+1)(x-1)}$$

$$= \frac{(x^3-x^2)}{(x+1)(x-1)} - \frac{(x+1)}{(x+1)(x-1)}$$

$$= \frac{(x^3-x^2) - 1(x+1)}{(x+1)(x-1)}$$

$$= \frac{x^3-x^2-x-1}{(x+1)(x-1)}, x \neq -1, 1$$

Example 4:

$$\frac{3}{x+5} - \frac{1}{4x+20}$$ LCD: $4(x+5)$

$$= \frac{3}{(x+5)} - \frac{1}{4(x+5)}$$

$$= \frac{4 \cdot 3}{4 \cdot (x+5)} - \frac{1}{4(x+5)}$$

$$= \frac{12}{4(x+5)} - \frac{1}{4(x+5)}$$

$$= \frac{12-1}{4(x+5)}$$

$$= \frac{11}{4(x+5)}, x \neq -5$$

Example 5:

$\frac{3}{2x}+\frac{4}{(x-1)}$ LCD: $2x(x-1)$

$= \frac{(x-1)}{(x-1)} \cdot \frac{3}{2x} + \frac{2x}{2x} \cdot \frac{4}{(x-1)}$

$= \frac{(3x-3)}{2x(x-1)} + \frac{8x}{2x(x-1)}$

$= \frac{3x-3+8x}{2x(x-1)}$

$= \frac{11x-3}{2x(x-1)}, x \neq 0, 1$

Example 6:

$$\frac{7}{x^2-9}+\frac{1}{4x+12}$$ LCD: $4(x+3)(x-3)$

$$= \frac{7}{(x+3)(x-3)} + \frac{1}{4(x+3)}$$

$$= \frac{4\cdot 7}{4\cdot(x+3)(x-3)} + \frac{(x-3)\cdot 1}{(x-3)\cdot 4(x+3)}$$

$$= \frac{28}{4(x+3)(x-3)} + \frac{(x-3)}{4(x+3)(x-3)}$$

$$= \frac{28+x-3}{4(x+3)(x-3)}$$

$$= \frac{x+25}{4(x+3)(x-3)}, x\neq -3, 3$$

Example 7:

$$\frac{(3x-2)}{(x+2)(x-2)} - \frac{(2x-4)}{(x+2)(x-2)}$$

$$= \frac{(3x-2) - 1\cdot(2x-4)}{(x+2)(x-2)}$$

$$= \frac{3x-2-2x+4}{(x+2)(x-2)}$$

$$= 1\cdot\frac{(x+2)}{(x+2)(x-2)}$$

$$= \frac{1}{x-2}, \; x \neq -2, 2$$

Example 8:

$\frac{2x}{x^2-1} - \frac{4}{x-1}$ LCD: $(x+1)(x-1)$

$= \frac{2x}{(x+1)(x-1)} - \frac{4}{(x-1)}$

$= \frac{2x}{(x+1)(x-1)} - \frac{(x+1)}{(x+1)} \cdot \frac{4}{(x-1)}$

$= \frac{2x}{(x+1)(x-1)} - \frac{(4x+4)}{(x+1)(x-1)}$

$= \frac{2x-4x-4}{(x+1)(x-1)}$

$= \frac{-2x-4}{(x+1)(x-1)}$

$= \frac{-2(x+2)}{(x+1)(x-1)}, x \neq -1, 1$

Your turn:

1. Simplify the following rational expressions:

(A)

$$\frac{x+7}{2x+14} - \frac{5x}{-3x-21}$$

$$= \frac{(x+7)}{2(x+7)} - \frac{5x}{-3(x+7)}$$

$$= \frac{(x+7)}{2(x+7)} - \frac{(-5x)}{3(x+7)} \quad \text{LCD: } 2\cdot 3(x+7)$$

$$= \frac{3\cdot(x+7)}{3\cdot 2(x+7)} - \frac{2\cdot(-5x)}{2\cdot 3(x+7)}$$

$$= \frac{3x+21}{2\cdot 3(x+7)} - \frac{(-10x)}{2\cdot 3(x+7)}$$

$$= \frac{3x+21+10x}{2\cdot 3(x+7)}$$

$$= \frac{13x+21}{6(x+7)}, \quad x \neq -7$$

(B)

$$\frac{5x-1}{2x+6}+\frac{16x}{x^2-9}$$

LCD: $2(x+3)(x-3)$

$$=\frac{(5x-1)}{2(x+3)}+\frac{16x}{(x+3)(x-3)}$$

$$=\frac{(x-3)}{(x-3)}\cdot\frac{(5x-1)}{2(x+3)}+\frac{2\cdot 16x}{2\cdot(x+3)(x-3)}$$

$$=\frac{5x^2-x-15x+3}{2(x+3)(x-3)}+\frac{32x}{2(x+3)(x-3)}$$

$$=\frac{5x^2-16x+3+32x}{2(x+3)(x-3)}$$

$$=\frac{5x^2+16x+3}{2(x+3)(x-3)},\ x\neq\pm 3$$

(C)

$$\frac{2x+3}{3x-3} - \frac{5x+3}{3x^2-3x}$$

$$= \frac{(2x+3)}{3(x-1)} - \frac{(5x+3)}{3x(x-1)} \quad \text{LCD: } 3x(x-1)$$

$$= \frac{x \cdot (2x+3)}{x \cdot 3(x-1)} - \frac{(5x+3)}{3x(x-1)}$$

$$= \frac{2x^2+3x}{3x(x-1)} - \frac{(5x+3)}{3x(x-1)}$$

$$= \frac{2x^2+3x-5x-3}{3x(x-1)}$$

$$= \frac{2x^2-2x-3}{3x(x-1)}, x \neq 0, 1$$

Textbook Questions: page 249 – 250 #1(a,c), 3(a,b), 4(a,b), 5(a,b), 6(a,c), 7, 8, 9, 10