Math 3201

4.5A Rational Equations

Now that we've learned to work with rational expressions, we can now piece them together and work with rational equations. A **rational equation** involves one or more rational expression. For example:

$$\frac{5}{x} = \frac{4}{x+2}$$

There are several methods that can be used for solving rational equations:

1. **Cross Multiplication**: can ONLY be used when there is a single fraction on each side of the equation.

2. **Common Denominator**: get a common denominator for each fraction, then drop the denominator and solve the equation formed by the numerators.

3. **Eliminate the Fractions**: multiply each term by the LCD. This will result in the denominators of each fraction being cancelled out.

We can use the techniques of multiplying/dividing and adding/subtracting that we learned earlier in this chapter to solve an equation for a variable.

Of the three methods, the third one is probably the most general and efficient, so we will go with that one.

Eliminating the Fractions

Steps:

- Find the LCD.
- Multiply each term by the LCD.
- Cancel terms that are common to the numerator and denominator of each fraction.
- Solve the resulting equation. If the equation is quadratic, you might need to factor or use the quadratic formula.
- Check each solution to determine if there are any extraneous roots.

Example 1:
Solve the following equation for x:

(A)

$$\frac{x}{10} = \frac{2}{5}$$ LCD: $10\cdot 5$

$$\cancel{10}\cdot 5\cdot \frac{x}{\cancel{10}} = 10\cdot\cancel{5}\cdot\frac{2}{\cancel{5}}$$

$$5\cdot x = 10\cdot 2$$

$$\frac{5x}{5} = \frac{20}{5}$$

$$x = 4$$

(B)

$$\frac{3}{x} + \frac{7}{2x} = \frac{1}{5}$$ LCD: $2\cdot 5x$

$$2\cdot 5\cdot\cancel{x}\cdot\frac{3}{\cancel{x}} + \cancel{2}\cdot 5\cdot\cancel{x}\cdot\frac{7}{\cancel{2}\cancel{x}} = 2\cdot\cancel{5}x\cdot\frac{1}{\cancel{5}}$$

$$30 + 35 = 2\cdot x$$

$$\frac{65}{2} = \frac{2x}{2}$$

$$x = \frac{65}{2} = 32.5,\ x \neq 0$$

(C)

$$\frac{x}{2}-\frac{(x+5)}{4}=\frac{4}{3}$$

CD: $2\cdot 3\cdot 4$

$$\cancel{2}\cdot 3\cdot 4\cdot\frac{x}{\cancel{2}}-2\cdot 3\cdot\cancel{4}\,\frac{(x+5)}{\cancel{4}}=2\cdot\cancel{3}\cdot 4\cdot\frac{4}{\cancel{3}}$$

$$12x-6(x+5)=32$$

$$12x-6x-30=32$$

$$6x=32+30$$

$$\frac{6x}{6}=\frac{62}{6}$$

$$x=\frac{31}{3}$$

(D)

$$\frac{1}{4}+\frac{1}{5}=\frac{1}{x}$$

LCD: $4\cdot 5\cdot x$

$$\cancel{4}\cdot 5x\cdot\frac{1}{\cancel{4}}+4\cdot\cancel{5}\cdot x\cdot\frac{1}{\cancel{5}}=4\cdot 5\cdot\cancel{x}\cdot\frac{1}{\cancel{x}}$$

$$5x+4x=20$$

$$\frac{9x}{9}=\frac{20}{9}$$

$$x=20/9\ ,\ x\neq 0$$

(E)

$$\frac{3}{x} - \frac{2}{4x} = \frac{1}{12}$$

LCD: $4 \cdot 12 \cdot x$

$$4 \cdot 12 \cdot \cancel{x} \cdot \frac{3}{\cancel{x}} - \cancel{4} \cdot 12 \cdot \cancel{x} \cdot \frac{2}{\cancel{4}\cancel{x}} = 4 \cdot \cancel{12} \cdot x \cdot \frac{1}{\cancel{12}}$$

$$144 - 24 = 4x$$

$$\frac{120}{4} = \frac{4x}{4}$$

$$x = 30, \; x \neq 0$$

(F)

$$\frac{(2x+3)}{(x+5)} + \frac{1}{2} = \frac{(-14)}{2(x+5)} \qquad LCD: 2(x+5)$$

$$\cancel{2(x+5)} \cdot \frac{(2x+3)}{\cancel{(x+5)}} + \cancel{2}(x+5) \cdot \frac{1}{\cancel{2}} = \cancel{2(x+5)} \frac{(-14)}{\cancel{2(x+5)}}$$

$$2(2x+3) + x + 5 = -14$$

$$4x + 6 + x + 5 = -14$$

$$5x = -14 - 6 - 5$$

$$\frac{5x}{5} = \frac{-25}{5}$$

$$x = -5, \quad x \neq -5$$

No solution.

(G)

$$\frac{(2x^2+1)}{(x+3)} = \frac{x}{4} + \frac{5}{(x+3)} \qquad \text{LCD: } 4(x+3)$$

$$\cancel{4(x+3)}\frac{(2x^2+1)}{\cancel{(x+3)}} = \cancel{4}(x+3)\cdot\frac{x}{\cancel{4}} + 4\cancel{(x+3)}\cdot\frac{5}{\cancel{(x+3)}}$$

$$4(2x^2+1) = x(x+3) + 20$$

$$8x^2+4 = x^2+3x+20$$

$$8x^2-x^2-3x+4-20=0$$

$$7x^2-3x-16=0$$

$$x = \frac{-(-3)\pm\sqrt{(-3)^2-4(7)(-16)}}{2(7)}$$

$$x = \frac{3\pm\sqrt{457}}{14}$$

$$x = \frac{3\pm 21.4}{14}$$

$$x = \frac{3+21.4}{14}, \quad x = \frac{3-21.4}{14}, \quad x \neq 3$$

$$x = 1.7, \quad x = -1.3$$

(H)

$$\frac{18}{x^2-3x}=\frac{6}{x-3}-\frac{5}{x}$$

$$\frac{18}{x(x-3)}=\frac{6}{(x-3)}-\frac{5}{x} \quad LCD: x(x-3)$$

$$\cancel{x}\cancel{(x-3)}\cdot\frac{18}{\cancel{x}\cancel{(x-3)}}=x\cancel{(x-3)}\cdot\frac{6}{\cancel{(x-3)}}-\cancel{x}(x-3)\cdot\frac{5}{\cancel{x}}$$

$$18=6x-(5x-15)$$

$$18=6x-5x+15$$

$$18-15=6x-5x$$

$$x=3\ ,\ x\neq 0,3$$

No solution.

(I)

$$\frac{2}{(a+2)}-\frac{(a^2+4)}{(a^2-4)}=\frac{a}{(2-a)}$$

$$\frac{a}{(2-a)} = \frac{a}{(-a+2)} = \frac{a}{-(a-2)} = \frac{-a}{(a-2)}$$

$$\frac{2}{(a+2)} - \frac{(a^2+4)}{(a+2)(a-2)} = \frac{-a}{(a-2)} \quad LCD: (a+2)(a-2)$$

$$\cancel{(a+2)}(a-2)\cdot\frac{2}{\cancel{(a+2)}} - \cancel{(a+2)}\cancel{(a-2)}\frac{(a^2+4)}{\cancel{(a+2)}\cancel{(a-2)}} = (a+2)\cancel{(a-2)}\frac{(-a)}{\cancel{(a-2)}}$$

$$2(a-2)-(a^2+4)=(-a)(a+2)$$

$$2a-4-\cancel{a^2}-4=-\cancel{a^2}-2a$$

$$2a+2a-8=0$$

$$\frac{4a}{4}=\frac{8}{4}$$

$$a=2, \quad a\neq\pm 2$$

No solution.

Textbook Questions: page 258 #1, 5, 6