Math 3201

4.5B Applications of Rational Equations

Here we will write rational equations to represent a variety of situations, and use them to solve for a variable. Once we solve for the variable, we not only need to check for extraneous roots, but we also need to consider inadmissible roots.

Inadmissible Roots

Values for the variable that do not make sense in the context of an application problem. For example, if we are solving for x and x represents time, then a negative x-value would be an inadmissible root since we cannot have negative time values. Also if a solution happens to be a non-permissible value, it is inadmissible. If this is the only solution, we say the problem has no solution.

Example 1:

~~When three more than a number is divided by twice the number~~, the result is the same as the original number. Find all numbers that satisfy these conditions.

X: unknown number

$$\frac{(x+3)}{2x} = x \qquad LCD: 2x$$

$$\cancel{2x} \cdot \frac{(x+3)}{\cancel{2x}} = 2x \cdot x$$

$$x+3 = 2x^2$$

$$0 = 2x^2 - x - 3$$

or

$$\underset{a}{2}x^2 \underset{b}{-} x \underset{c}{-3} = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{1 \pm \sqrt{25}}{4}$$

$$x = \frac{1 \pm 5}{4}$$

$$x = \frac{1+5}{4}, \quad x = \frac{1-5}{4}$$

$$x = \frac{6}{4}, \quad x = \frac{-4}{4}$$

$$x = \frac{3}{2}, \quad x = -1, \quad x \neq 0$$

Example 2:

One positive integer is 5 more than the other. When the reciprocal of the larger number is subtracted from the reciprocal of the smaller number, the result is $\frac{5}{14}$. Find the two integers.

x: positive integer

x+5: other positive integer

$$\frac{1}{x} - \frac{1}{(x+5)} = \frac{5}{14} \quad \text{LCD: } 14x(x+5)$$

$$14\cancel{x}(x+5)\cdot\frac{1}{\cancel{x}} - 14x\cancel{(x+5)}\cdot\frac{1}{\cancel{(x+5)}} = \cancel{14}x(x+5)\cdot\frac{5}{\cancel{14}}$$

$$\cancel{14x}+70-\cancel{14x} = 5x^2+25x$$

$$\frac{0}{5} = \frac{5x^2+25x-70}{5}$$

$$0 = x^2+5x-14$$

$$0 = (x+7)(x-2)$$

$$\cancel{x=-7},\ \boxed{x=2},\ x \neq 0, -5$$

Work Rate Problem

We have two equivalent formulae to choose from when solving a work rate problem. If two people are working together on a job, their rates add and they can perform the job in shorter time.

If we let x = time it takes person 1 to complete the job, then the work rate is $\frac{1}{x}$. In other words, this person can complete the job in x hours.

If we let y = time it takes person 2 to complete the job, and t = time i~~s~~t takes with both working together, we get the following formulas:

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{t}$$

Example 3:

Sherry mows a lawn in 4 hours. Mary mows the same lawn in 5 hours. How long would it take both of them working together to mow the lawn? * Complete following table

Name	time to mow	fraction mowed in 1h.
Sherry	4	$\frac{1}{4}$
Mary	5	$\frac{1}{5}$
both	x	$\frac{1}{x}$

equation

$r_1 + r_2 = r_3$

$$\frac{1}{4}+\frac{1}{5}=\frac{1}{x}$$

LCD: $4\cdot5\cdot x$

$$\cancel{4}\cdot5x\cdot\frac{1}{\cancel{4}}+4\cdot\cancel{5}x\cdot\frac{1}{\cancel{5}}=4\cdot5\cdot\cancel{x}\cdot\frac{1}{\cancel{x}}$$

$$5x+4x=20$$

$$\frac{9x}{9}=\frac{20}{9}$$

$\rightarrow x = 2.2\text{ h}$

It takes the girls 2.2h to mow the lawn together.

Example 4:
It takes Mike 9 hours longer to construct a fence than it takes Jason. If they work together, they can construct the fence in 20 hours. How long would it take Mike to construct the fence alone?

Name	time	fence built in 1 hr
Mike	$x+9$	$\frac{1}{x+9}$
Jason	x	$\frac{1}{x}$
both	20	$\frac{1}{20}$

$$\frac{1}{(x+9)} + \frac{1}{x} = \frac{1}{20} \quad \text{LCD: } 20x(x+9)$$

$$20x\cancel{(x+9)}\cdot\frac{1}{\cancel{(x+9)}} + 20\cancel{x}(x+9)\cdot\frac{1}{\cancel{x}} = \cancel{20}x(x+9)\cdot\frac{1}{\cancel{20}}$$

$$20x + 20x + 180 = x^2 + 9x$$

$$0 = x^2 - 31x - 180$$

$$x = \frac{-(-31) \pm \sqrt{(-31)^2 - 4(1)(-180)}}{2(1)}$$

$$x = \frac{31 \pm \sqrt{1681}}{2}$$

$$x = \frac{31 \pm 41}{2}$$

$$\to x = \frac{31+41}{2}, \quad x = \frac{31-41}{2}$$

$$x = \frac{72}{2}, \quad x = \frac{-10}{2}$$

$$\boxed{x = 36}, \quad \cancel{x = -5}$$

Jason: 36 hrs

Mike: 36+9 = 45 hrs

Example 5:

Vanessa bought a case of concert t-shirts for $450. She kept two t-shirts for herself and sold the rest for $560, making a profit of $10 on each t-shirt. How many t-shirts were in the case? Use the following equation: $\frac{560}{(x-2)} - \frac{450}{x} = 10$ LCD: $x(x-2)$

$$x\cancel{(x-2)} \cdot \frac{560}{\cancel{(x-2)}} - \cancel{x}(x-2) \cdot \frac{450}{\cancel{x}} = x(x-2)10$$

$$560x - 450x + 900 = 10x^2 - 20x$$

$$110x + 900 = 10x^2 - 20x$$

$$\frac{0}{10} = \frac{10x^2 - 130x - 900}{10}$$

$$x^2 - 13x - 90 = 0$$

$$(x+5)(x-18) = 0$$

$$\cancel{x = -5}, \boxed{x = 18}$$

There are 18 t-shirts in the case.

Example 6:
When they work together, Stuart and Lucy can deliver flyers to all the homes in their neighbourhood in 42 minutes. When Lucy works alone, she can deliver the flyers in 13 minutes less than Stuart when he works alone. When Stuart works alone, how long does he take to deliver the flyers?

Name	time	fraction delivered in 1 minute
Stuart	x	$\frac{1}{x}$
Lucy	$x-13$	$\frac{1}{x-13}$
both	42	$\frac{1}{42}$

$$\frac{1}{x} + \frac{1}{(x-13)} = \frac{1}{42} \quad \text{LCD: } 42x(x-13)$$

$$42\cancel{x}(x-13)\cdot\frac{1}{\cancel{x}} + 42x\cancel{(x-13)}\cdot\frac{1}{\cancel{(x-13)}} = \cancel{42}x(x-13)\cdot\frac{1}{\cancel{42}}$$

$$42x - 546 + 42x = x^2 - 13x$$

$$0 = x^2 - 13x - 84x + 546$$

$$0 = x^2 - 97x + 546$$

$$(x-91)(x-6) = 0$$

$$\boxed{x = 91}, \cancel{x = 6}$$

It takes Stuart 91 minutes to deliver the flyers.

Example 7:
Erica frequently drives 189 km to visit friends in St. John's. She has noticed that she saves 36 minutes if she travels 24 km/h faster than her average speed. What is her average speed? Use the following equation: $\frac{189}{x} - \frac{189}{x+24} = \frac{3}{5}$. LCD: $5x(x+24)$

$$\cancel{5x}(x+24)\cdot\frac{189}{\cancel{x}} - 5x\cancel{(x+24)}\cdot\frac{189}{\cancel{(x+24)}} = \cancel{5}x(x+24)\cdot\frac{3}{\cancel{5}}$$

$$\cancel{945x} + 22680 - \cancel{945x} = 3x^2 + 72x$$

$$\frac{0}{3} = \frac{3x^2 + 72x - 22680}{3}$$

$$0 = x^2 + 24x - 7560$$

$$x = \frac{-24 \pm \sqrt{(24)^2 - 4(1)(-7560)}}{2(1)}$$

$$x = \frac{-24 \pm \sqrt{30816}}{2}$$

$$x = \frac{-24 \pm 175.5}{2}$$

$$x = \frac{-24 + 175.5}{2}, \quad x = \frac{-24 - 175.5}{2}$$

$$\boxed{x = 75.8 \text{ km/h}} \qquad \cancel{x = -199.5 \text{ km/h}}$$

Textbook Questions: page 259, 260 #8, 10, 11, 12, 13, 14, 15, 16, 17