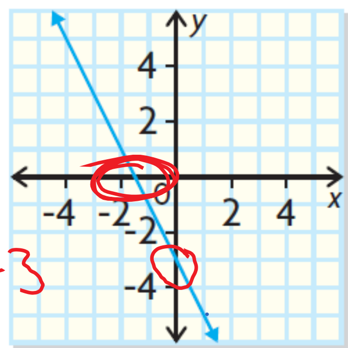
Math 3201

5.2 Characteristics of the Equations of Polynomials

Reading the Y-Intercept of a Function from the Equation

The y-intercept corresponds to the constant term in the equation. This applies to all types of polynomial functions.

Y-int: -3



$f(x) = -2x - 3$

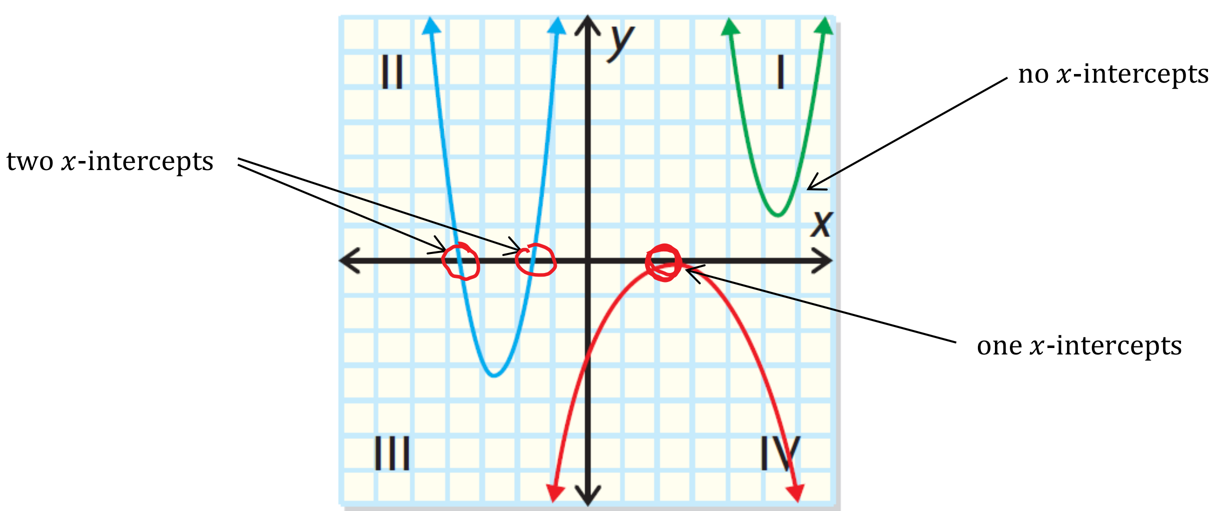
The constant term in the equation is ~~3~~ -3
and the y-intercept of the graph is ~~3~~ -3

Recall from Math 1201, given $y = mx + b$, m is the slope of the line. $m = \frac{\text{rise}}{\text{run}}$. The further from 0 a slope is, the steeper it is on a graph. So for this example, slope $= m = -2$

Determining the Maximum number of X-Intercepts of a Function from the Equation

The maximum number of x-intercepts for a function is equal to the degree of the function. For example, a linear function can have a maximum of 1 x-intercept and the degree of a linear function is 1. The example, above, shows that we have an x-intercept of $x = -\frac{3}{2}$ or -1.5.

For **quadratics**, the degree is 2, so the maximum number of x-intercepts is 2. However, a quadratic can have less than two x-intercepts.



Thus, a quadratic can have 0, 1 or 2 x-intercepts. The degree tells us the **maximum** number of x-intercepts a function can have. Recall from Math 2201, the vertex (h, k) of a parabola is the maximum or minimum value.

If $f(x) = ax^2 + bx + c$,

$$h = -\frac{b}{2a} \text{ and } k = f(h)$$

The range of quadratic is $\{y|y \geq k, y \in R\},\ a > 0$ and $\{y|y \leq k, y \in R\},\ a < 0$

We can also use a, h and k to write a quadratic function in vertex form:

$$f(x) = a(x-h)^2 + k$$

Example 1:

(A)

i. What is $y = -3x^2 - 12x + 7$ in vertex form? $\boxed{y = a(x-h)^2 + k}$

$a = -3$

$h = -\frac{b}{2a} = -\frac{(-12)}{2(-3)} = \frac{12}{-6} = -2$

$k = -3(-2)^2 - 12(-2) + 7 = 19$

$\therefore y = -3(x-(-2))^2 + 19$

$y = -3(x+2)^2 + 19$

Vertex: $(-2, 19)$

ii. What is the range?

$a < 0$ $\{y|y \leq k, y \in R\}$ $\therefore \{y|y \leq 19, y \in R\}$

(B)

i. What is $y = 2x^2 - 16x - 11$ in vertex form?

$a = 2$

$h = -\frac{b}{2a} = -\frac{(-16)}{2(2)} = \frac{16}{4} = 4$

$k = 2(4)^2 - 16(4) - 11 = -43$

$\therefore y = 2(x-4)^2 - 43$

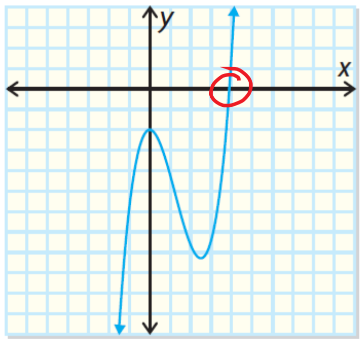
Vertex: $(4, -43)$

ii. What is the range?

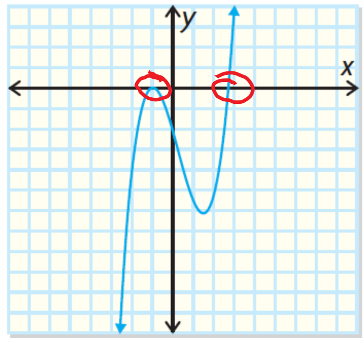
$a > 0$ $\{y|y \geq k, k \in R\}$ $\therefore \{y|y \geq -43, y \in R\}$

Cubic functions have a degree of 3, thus they have a maximum of 3 x-intercepts.
Below are examples of cubic functions having two turning points:

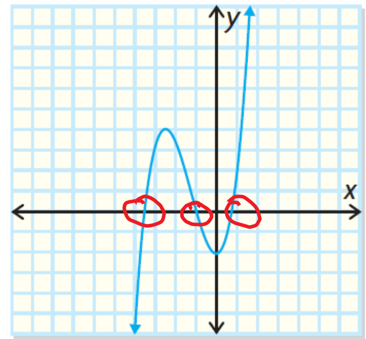
One x-intercept:



Two x-intercepts:

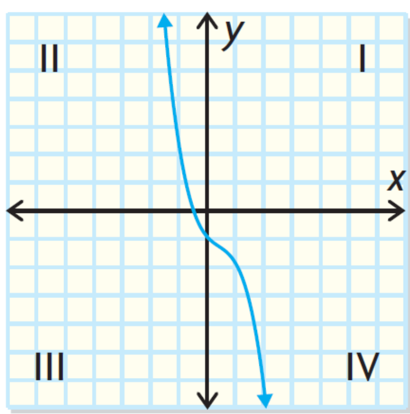


Three x-intercepts:



Thus, cubic functions having two turning points can have 1, 2 or 3 x-intercepts.

A Special Case



Cubic functions that have no turning points will only have 1 x-intercept.

Matching Equations and Graphs
Steps:

- Look at the degree of the equation to determine what type of function it is.
- Match the constant term in the equation with the y-intercept on the graph.
- Check the leading coefficient of the equation and match it up with the end behavior of the graph.

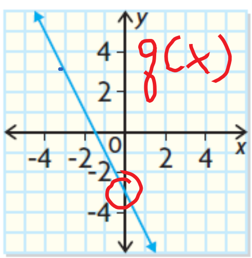
Example 2:

Match each graph with the correct polynomial function.
Justify your reasoning.

vi $g(x) = -x^3 + 4x^2 - 2x - 2$ ii $j(x) = x^2 - 2x - 2$ iv $p(x) = x^3 - 2x^2 - x - 2$

iii $h(x) = -\frac{1}{2}x - 3$ v $k(x) = x^2 - 2x + 1$ i $q(x) = -2x - 3$

i)



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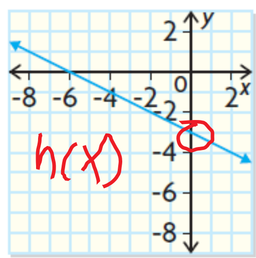

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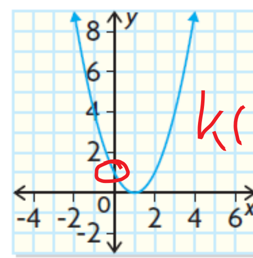

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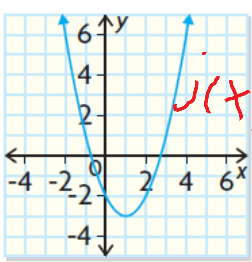
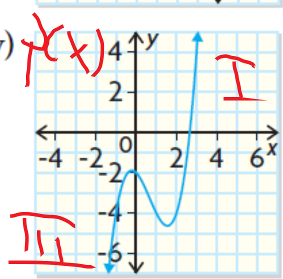
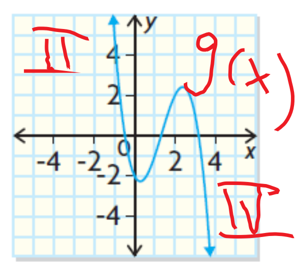


iv)



vi)

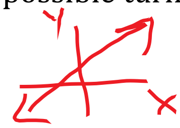


Example 3:

Determine the following characteristics of each function using the equation:

- y-intercept
- end behavior
- domain
- range
- number of possible turning points

(A) $f(x) = 3x - 5$



y-int: -5

end behavior: fall left / rise right Q III → Q I

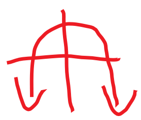
domain: $\{x \mid x \in \mathbb{R}\}$

range: $\{y \mid y \in \mathbb{R}\}$

turning points: 0

(B) $f(x) = -2x^2 - 4x + 8$

a b c



y-int: 8

end behavior: falls left / falls right Q III → Q IV

domain: $\{x \mid x \in \mathbb{R}\}$

range: $h = -\frac{b}{2a} = -\frac{(-4)}{2(-2)} = \frac{4}{-4} = -1$ $\{y : y \le 10, y \in \mathbb{R}\}$

$k = -2(-1)^2 - 4(-1) + 8 = 10$ turning points: 1

(C) $f(x) = 2x^3 + 10x^2 - 2x - 10$

y-int: -10

end behavior: $a > 0$ falls left / rises right Q III → Q I

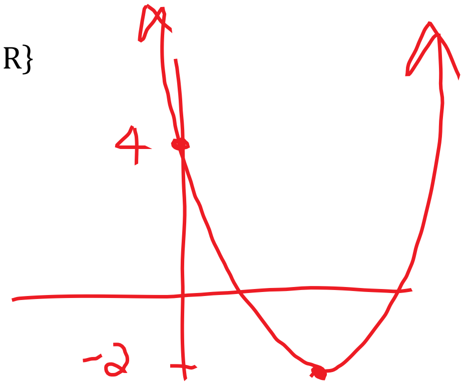
domain: $\{x \mid x \in \mathbb{R}\}$

range: $\{y \mid y \in \mathbb{R}\}$ turning points: 2 or 0

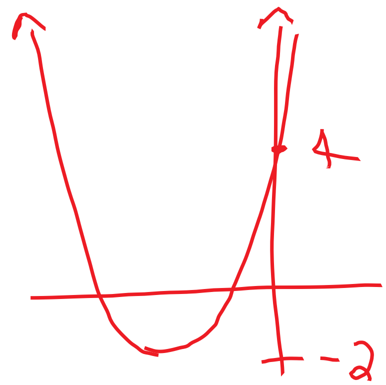
Example 4:
Sketch the graph of a possible polynomial function for each set of given characteristics.

(A) range: $\{y | y \geq \underline{-2}, y \in R\}$
y-intercept: 4

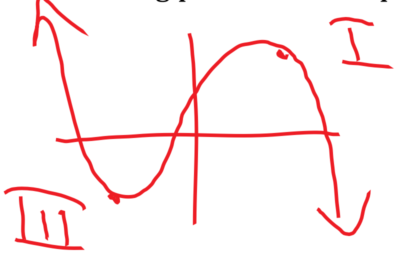
parabola
• opens up



or

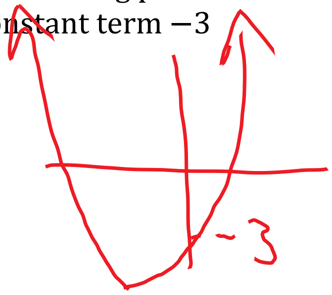


(B) range: : $\{y | y \in R\}$
turning points: one in quadrant III and another in quadrant I



(C) two turning points
negative leading coefficient
constant term -5

(D) degree 2
one turning point which is a minimum
constant term -3



Textbook Questions: page 287 - 289 #1, 2, 3, 6, 7, 13