Math 3201

6.1 Exploring the Characteristics of Exponential Functions

Exponential Functions

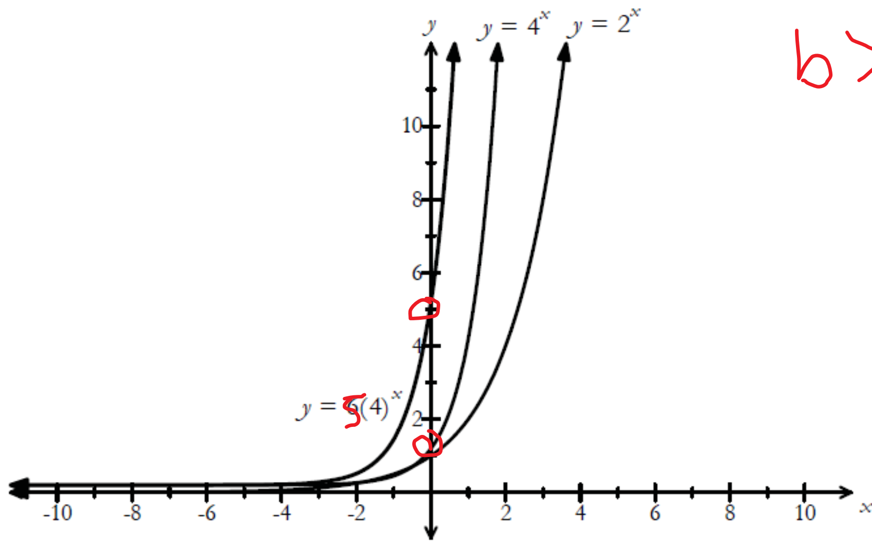
Equations written in the form:

$$y = a(b)^x$$

where:

- $b > 0$ and $b \neq 1$
- $a > 0$ for the cases that we will study
- x is the exponent instead of the base, as it was for the other functions we looked at

We will now explore what the graphs of exponential functions look like, and examine some basic properties of the graphs. Consider the following graph, and use it complete the table that follows:

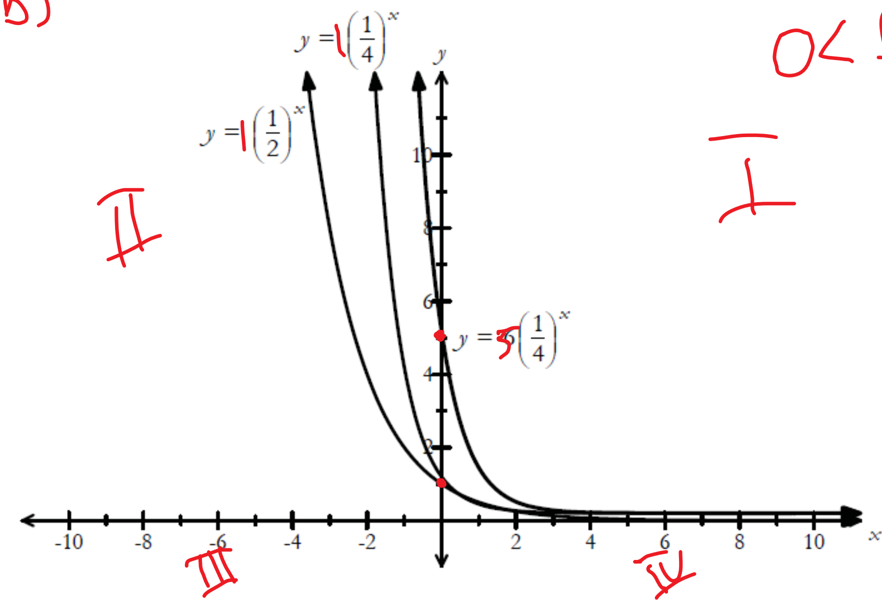


$b > 1$

	$y = 2^x$	$y = 4^x$	$y = \cancel{6}5(4)^x$
y-intercept	1	1	5
number of x-intercepts	0	0	0
end behavior	increasing QII → QI	increasing QII → QI	increasing QII → QI
domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
range	$\{y \mid y > 0, y \in R\}$	$\{y \mid y > 0, y \in R\}$	$\{y \mid y > 0, y \in R\}$

$y = a(b)^x$

$0 < b < 1$



	$y = \left(\frac{1}{2}\right)^x$	$y = \left(\frac{1}{4}\right)^x$	$y = \cancel{6}5\left(\frac{1}{4}\right)^x$
y-intercept	1	1	5
number of x-intercepts	0	0	0
end behavior	decreasing QII → QI	decreasing QII → QI	decreasing QII → QI
domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
range	$\{y \mid y > 0, y \in R\}$	$\{y \mid y > 0, y \in R\}$	$\{y \mid y > 0, y \in R\}$

General Properties of $y = a(b)^x$

- no x-intercepts; one y-intercept
- exponential functions have a restricted range bounded by the x-axis but the domain consists of real numbers
- can be increasing or decreasing
- some exponential functions increase/decrease at a faster rate than others

Asymptotes

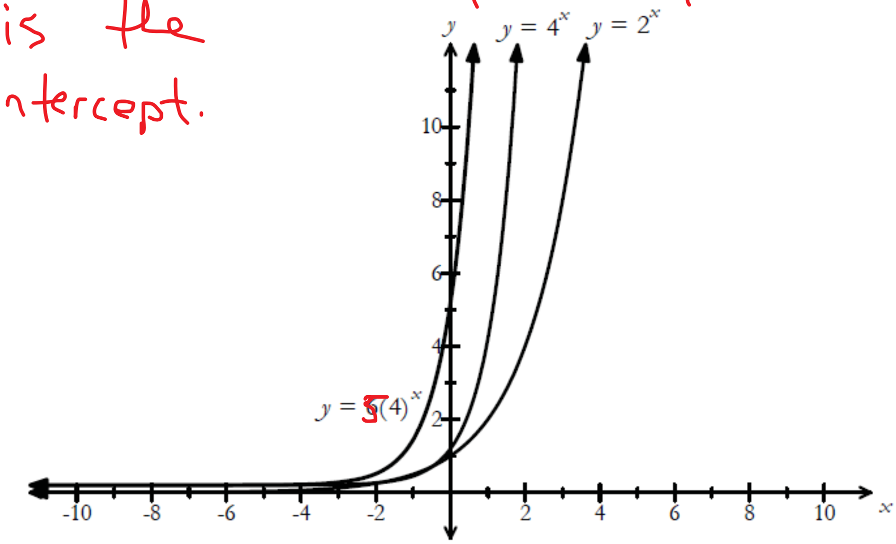
Notice that on each of our graphs, the exponential function got really close to the x-axis on one side, but never actually touched it. Thus, the x-axis is said to be an ***asymptote*** of the exponential function. More specifically, it is called a horizontal asymptote since it is a horizontal line. The x-axis has the equation $y = 0$, thus $y = 0$ is the asymptote of the exponential functions that we will study.

Y-Intercept of an Exponential Function

Consider the following graph again:

$y = 1(4)^x$ $y = 1(2)^x$

a is the y-intercept.



For each exponential function shown, identify the y-intercept from the graph, and state the value of a from the equation.

$y = 2^x \rightarrow y = 1(2)^x$ y-int: $a = 1$

$y = 4^x \rightarrow y = 1(4)^x$ y-int: $a = 1$

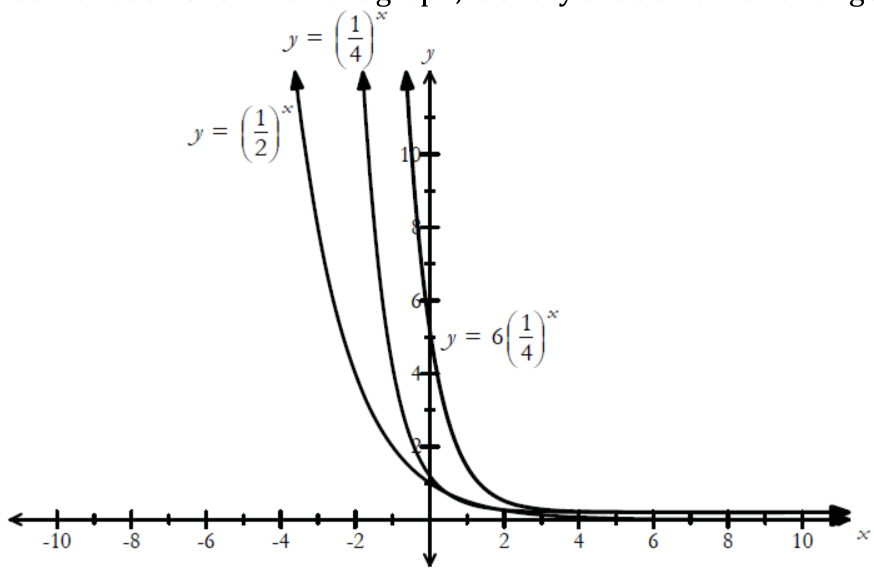
$y = 5(4)^x \longrightarrow$ y-int: $a = 5$

What is the relationship between the y-intercept and the value of a?

a is the y-intercept.

Domain and Range for Exponential Functions

For each exponential function shown on the graph, identify the domain and range:



Summary: For an exponential function written in the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$.

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y > 0, y \in R\}$

In Summary

Key Ideas

- An exponential function has the form $f(x) = a(b)^x$, where x is the exponent and $a \neq 0$, $b > 0$, and $b \neq 1$.
- All exponential functions of the form $f(x) = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, have the following characteristics:

Number of x-Intercepts	0
y-Intercept	a
End Behaviour	Curve extends from quadrant II to quadrant I.
Domain	$\{x \mid x \in R\}$
Range	$\{y \mid y > 0, y \in R\}$

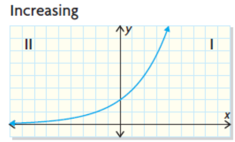
Need to Know

- There are two different shapes of the graphs of an exponential function of the form $f(x) = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$:

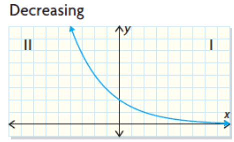
  - Case 1: An increasing function; the curve extends from quadrant II to quadrant I.

  - Case 2: A decreasing function; the curve extends from quadrant II to quadrant I.

Increasing



Decreasing



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