**Math 3201**

# 6.2 Relating the Characteristics of Exponential Functions to Equations

**Exponential Functions**

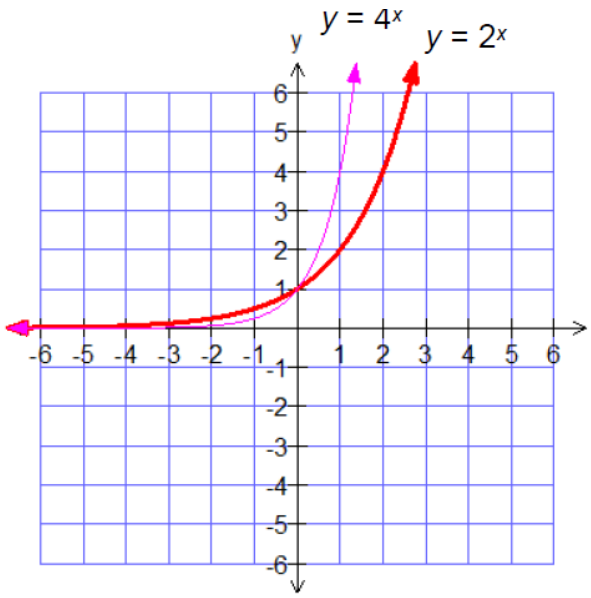
Equations written in the form:

$$y = a(b)^x$$

where:

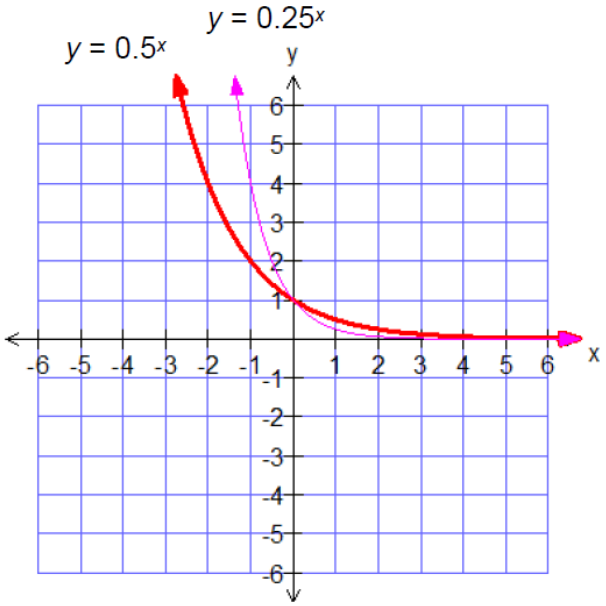
- $b > 0$ and $b \neq 1$
- $a > 0$ for the cases that we will study
- $x$ is the exponent instead of the base, as it was for the other functions we looked at

**Impact of *b* Value On the Graph**




Notice that for each function shown, b > 1, and that each of the graphs is increasing as we move from left to right.

Larger b, the steeper the graph to the right.




Notice that for each function shown, $0 < b < 1$, and that each of the graphs is decreasing as we move from left to right.

Smaller b, the steeper the graph to the left.

**Summary of Rules for *a* and *b* values**

For an exponential function of the form

$$y = a(b)^x$$

- a is the y-intercept on the graph
- if $b > 1$, the graph will increase
- if $0 < b < 1$, the graph will decrease

**Matching Equations With Graphs**

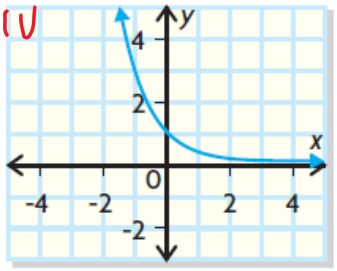
To match exponential equations with graphs, we must look at the *a* value in the equation and match it with the y-intercept on the graph. We must also look at the *b* value and determine whether the function is increasing or decreasing.

**Example 1:**

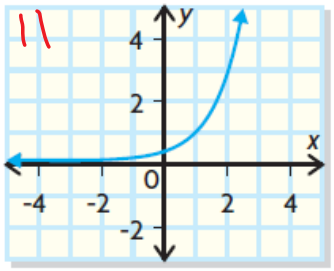
Match each function with the corresponding graph below. Provide your reasoning.

i) $y = (3)^x$ a=1 b=3 ii) $y = \frac{1}{3}(3)^x$ a=1/3 b=3 iii) $y = 3\left(\frac{1}{3}\right)^x$ a=3 b=1/3 iv) $y = 1\left(\frac{1}{3}\right)^x$ a=1 b=1/3

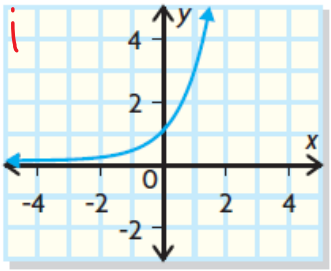
a) iv



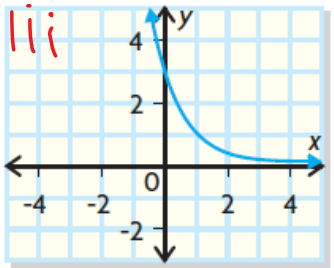

c) ii




b) i

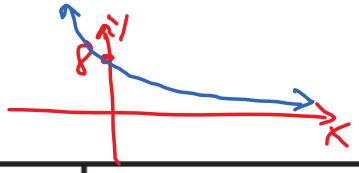



d) iii




**Example 2:**

Complete the following table for the function $y = 8\left(\frac{2}{3}\right)^x$




| $y = 8\left(\frac{2}{3}\right)^x$ | True | False | Why I think so |
|---|---|---|---|
| (i) the $y$-intercept is 1 | | ✓ | a=8 |
| (ii) the graph has one $x$-intercept | | ✓ | $y=ab^x$ have no x-int |
| (iii) the range is $\{y \mid y > 0, y \in R\}$ | ✓ | | For all $y=ab^x$ |
| (iv) the domain is $\{x \mid x > 8, x \in R\}$ | | ✓ | $\{x \mid x \in R\}$ |
| (v) this is a decreasing exponential function | ✓ | | $0<b<1$ |

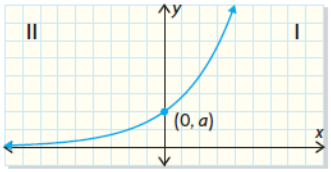
## In Summary

### Key Ideas

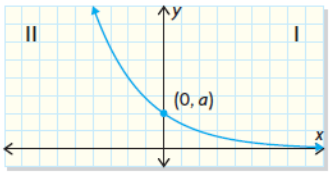
- In a table of values for an exponential function, there is a constant ratio between consecutive $y$-values when the $x$-values increase by the same amount. The value of this ratio is equal to the parameter $b$ in the function $y = a(b)^x$, where $b \neq 1$.
- In an exponential function of the form $y = a(b)^x$, $a$ is a non-zero multiplier and $b$ is the base (where $b > 0$ and $b \neq 1$). The value of $a$ is the $y$-intercept of the graph of the function.

### Need to Know

- An exponential function is an increasing function if $a > 0$ and $b > 1$.
- An exponential function is a decreasing function if $a > 0$ and $0 < b < 1$.
- Changing the parameters $a$ and $b$ in exponential functions of the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, does not change the number of $x$-intercepts, the end behaviour, the domain, or the range of the function. These characteristics are identical in all exponential functions of this form.



$a > 0, b > 1$



$a > 0, 0 < b < 1$

**Textbook Questions:** page 347 #2, 3, 4, 6, 7, 9, 10, 11, 12, 13

Homework