Math 3201

7.1 Introduction to Logarithmic Functions

Inverse of a Function: this is obtained by switching the values of x and y. The graph of the inverse of a function is reflected in the line $y = x$.

Complete the table of values for the exponential function $y = 10^x$ and its inverse $x = 10^y$.

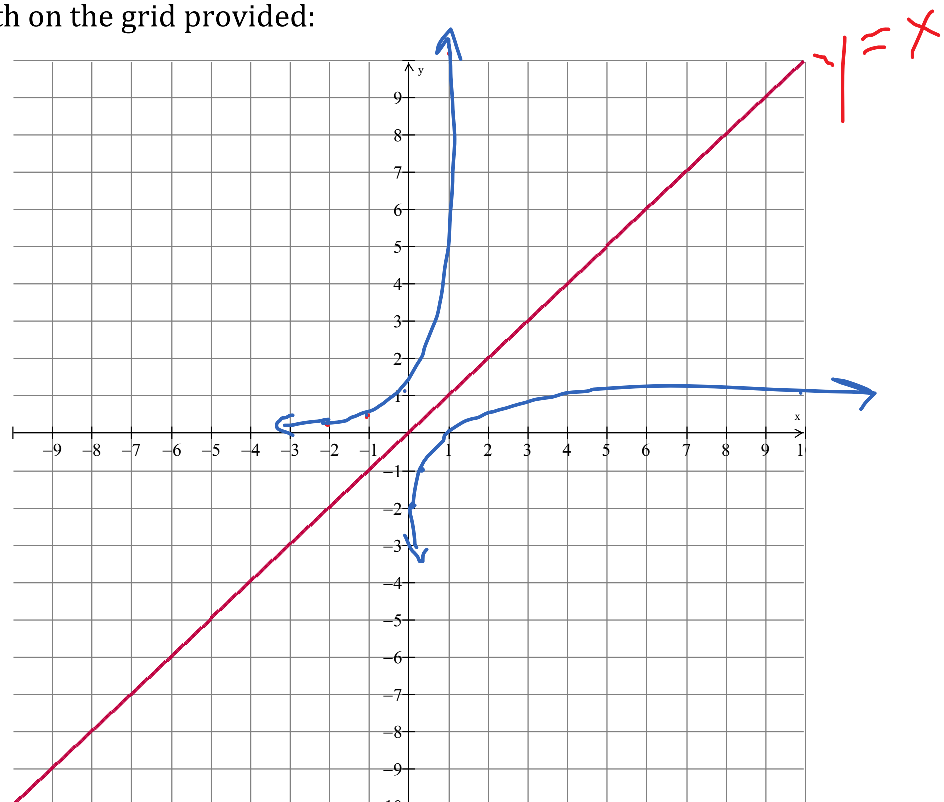
$y = 10^x$

x	y
-2	0.01
-1	0.1
0	1
1	10
2	100

$x = 10^y$

x	y
0.01	-2
0.1	-1
1	0
10	1
100	2

Graph both on the grid provided:



$y=10^x$ and its inverse $x=10^y$ are reflections of each other in the line $y=x$

Complete the following:

1. What is the relationship between the graphs of $y = 10^x$ and $x = 10^y$? Think about both graphs in relation to the line $y = x$?

Reflections of each other.

2. State the domain for each function.

Exponent: $D: \{x \mid x \in R\}$

Inverse: $D \{x \mid x > 0, x \in R\}$

3. State the range for each function.

Exponent: $\{y \mid y > 0, y \in R\}$

Inverse: $\{y \mid y \in R\}$

4. State the intercepts for each function.

$y = 10^x$ no x-intercept, y-intercept: 1

$x = 10^y$ x-intercept: 1, no y-intercept

5. Describe the end behavior for each function.

$y = 10^x$ left: decreases slowly
right: increases
Q II → Q I

$x = 10^y$ left: decreases
right: increases slowly
Q IV → Q I

The equation $y = 10^x$ represents an **exponential function.** The equation $x = 10^y$ represents the **inverse** of the exponential function.

Normally when we write an equation, we get "y" by itself, yet in the equation $x = 10^y$, "x " is by itself instead. To get "y" by itself in this equation, we write it in a different form called **logarithmic form.**

The equation $x = 10^y$ can also be written as:

$$y = \log_{10} x$$

This reads "log base 10 of x ", and it means: "What exponent would we have to raise 10 to in order to get x.

Note: The base of a logarithmic function can be some value other than 10, but 10 is the most common value. When we use the log button on a calculator, it automatically assumes a base of 10. Also, if we write a log equation without writing the base value: $y = \log x$ then it is automatically assumed that the base is 10. That is:

$$\log_{10} x = \log x$$

In general, the inverse of the exponential function $y = a(b)^x$ can be written as $x = a(b)^y$, or in **logarithmic form** as:

$$y = a\log_b x$$

"Ben" and "Benny"
A simple way to remember how to convert from exponent form to logarithmic form is:

$$b^e = n \qquad \log_b n = e$$

$$\underset{b}{10}^{\overset{e}{3}} = x_n \qquad \log_{10} x = 3$$

Comparing the Properties of Exponential and Logarithmic Functions

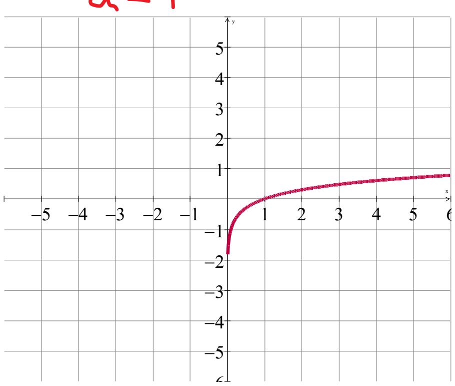
	Exponential	Logarithmic
Domain	all reals	positive x values
Range	positive y values	all reals
y-intercept	one y-intercept (0, 1)	no y-intercept
x-intercept	no x-intercept	one x-intercept (1, 0)
increasing/ decreasing	increasing from Quadrant II to Quadrant I	increasing from Quadrant IV to Quadrant I
end behaviour	as $x \to \infty$, $y \to \infty$ as $x \to -\infty$, $y \to 0$	as $x \to \infty$, $y \to \infty$ as $x \to 0$ on the right, $y \to -\infty$

Examining the Impact of the "a" value on a Logarithmic Function
Many logarithmic functions are written in the form $y = a\log_{10}x$. What impact does the value of "a" have on the appearance of the graph?

Consider the following graphs:

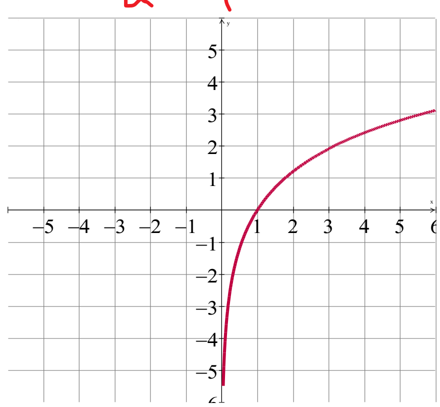
$y = \log x$

$a = 1$



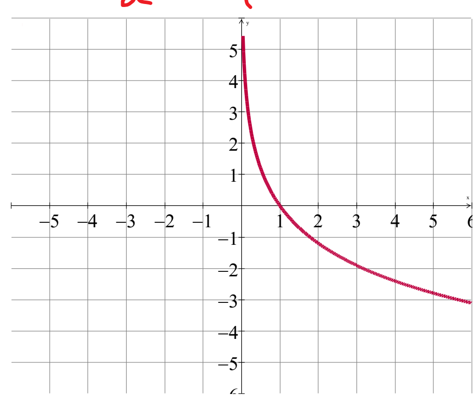
$y = 4\log x$

$a = 4$



$y = -4\log x$

$a = -4$



1. What is the impact on the graph of the function:

(A) if $a > 0$?

increasing

(B) if $a < 0$?

decreasing

2. Does a affect the x-coordinate or the y-coordinate? Is this a vertical transformation or a horizontal transformation?

"a" affects the y-coordinates

(flips upside down)

3. Which point is easily identified from the graph?

x-int: $x=1$

4. Which characteristics of the graphs of logarithmic functions differ from the characteristics of the graphs of exponential functions?

See previous table

Important Points:

- When $a > 0$, the y-values increase as the x-values increase. This is an increasing function from Quadrant IV to Quadrant I.
- When $a < 0$, y-values decrease as the x-values increase. This is a decreasing function from Quadrant I to Quadrant IV.
- Logarithmic functions do not have a y-intercept but do have a restricted domain, $x > 0$.

Question 1:

(A) What is the domain of the logarithmic function $y = \log x$?

D: $\{x \mid x > 0, x \in R\}$

log(−3)

(B) Use your calculator to evaluate ~~log(3)~~. What do you get? Explain why this is the case.

$\log(-3) = $ undefined

$x > 0$ ∴ can't take the log of a negative number.

(C) Use your calculator to evaluate log(0). What do you get? Explain why this is the case.

$\log(0) = $ undefined

$x > 0$ ∴ we can't take the log or 0.

Natural Logarithms
Natural logarithms are special case of logs in which the base is "e", where e is a constant, irrational number $e = 2.718281828...$

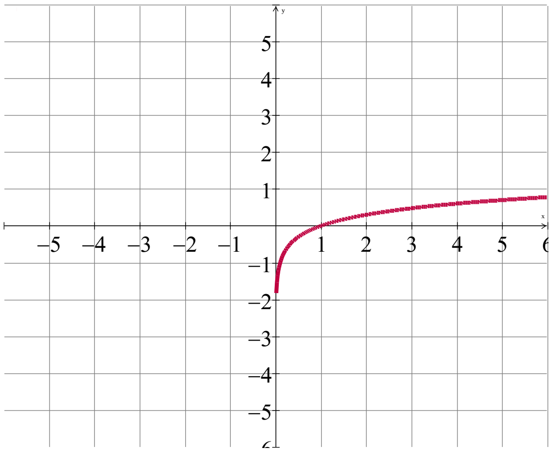
The constant "e" can be used as the base in an exponential function to give $y = a(e)^x$. The inverse would be $x = a(e)^y$. We can write this in logarithmic form as

$$y = a\log_e x \quad \text{or} \quad y = a\ln x$$

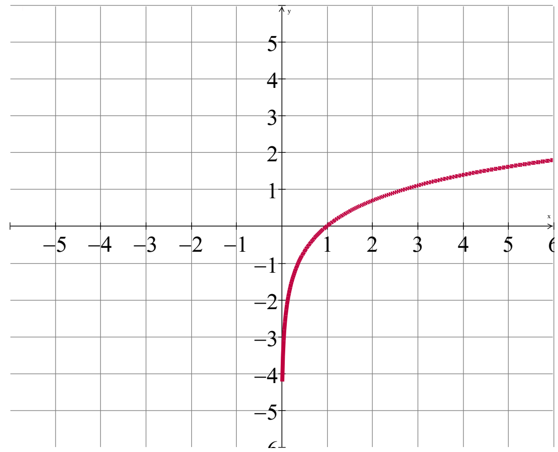
The latter format is the standard one for writing a logarithm with base e and is called the **natural logarithm**.

Comparing the Graphs of $y = \log_{10}x$ and $y = \ln x$

$y = \log_{10}x$



$y = \ln x$



Notice that changing the base of the logarithm from 10 to e slightly changes some of the y-values. However, the properties of the graphs are the same.

Key Ideas

- A logarithmic function has the form $f(x) = a \log_b x$, where $b > 0$, $b \neq 1$, and $a \neq 0$, and a and b are real numbers.
- All logarithmic functions of the form $f(x) = a \log x$ and $f(x) = a \ln x$ have these characteristics:

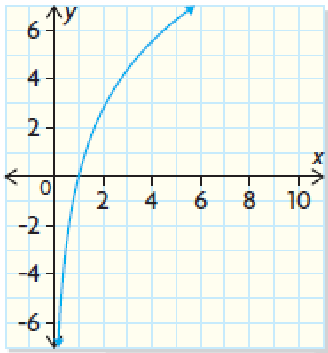
x-Intercept	1
Number of y-Intercepts	0
End Behaviour	The curve extends from quadrant IV to quadrant I or quadrant I to quadrant IV.
Domain	$\{x \mid x > 0, x \in R\}$
Range	$\{y \mid y \in R\}$

- All logarithmic functions of the form $f(x) = a \log x$ and $f(x) = a \ln x$ have these unique characteristics:
  - If $a > 0$, the function increases.
  - If $a < 0$, the function decreases.

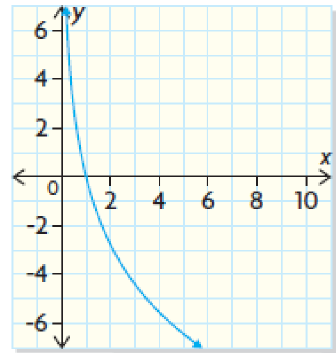
Need to Know

- The graph of a logarithmic function of the form $f(x) = a \log x$ or $f(x) = a \ln x$ will look like one of the following cases:

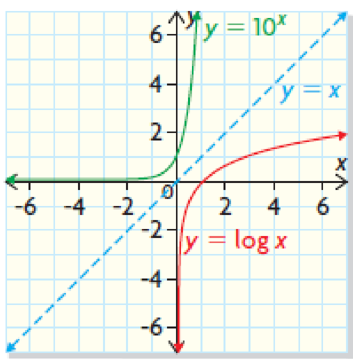
  Case 1: an increasing function, where $a > 0$



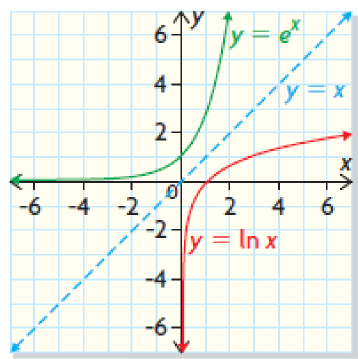
  Case 2: a decreasing function, where $a < 0$



- The graph of $y = \log x$ is a reflection of the graph of $y = 10^x$ about the line $y = x$.



- The graph of $y = \ln x$ is a reflection of the graph of $y = e^x$ about the line $y = x$.



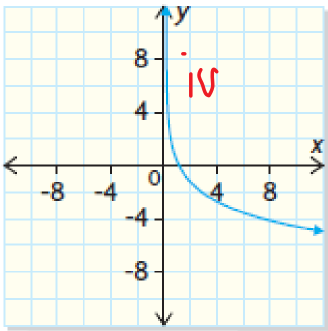
Matching Exponential and Logarithmic Equations with Graphs

Example 1:

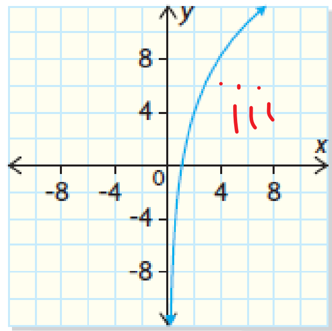
Which function matches each graph below? Provide your reasoning.

i) $y = 5(2)^x$ **ii)** $y = 2(0.1)^x$ **iii)** $y = 6 \log x$ **iv)** $y = -2 \ln x$

a)



c)



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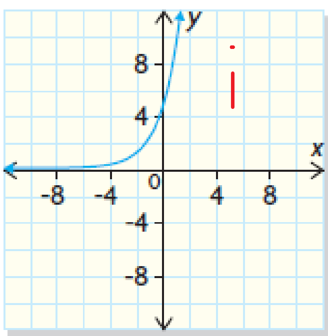

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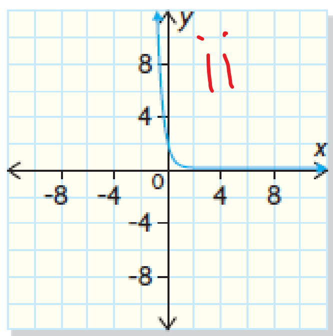

Textbook Questions: page 420 - 422 #1, 2, 5 a, b, d, e, 6, 7, 8