Math 3201

7.2 Evaluating Logarithmic Expressions

In Chapter 6 we evaluated exponential equations where we could express both sides of the equation as a common base and then work with the exponents. When we encountered equations where we could not express both sides as a common base we used our calculators and estimated the exponent through trial and error.

Now that we know exponents have inverses called logarithms we have a better way to evaluate all exponential equations.

The parts of an exponent are as follows:

exponent　　　argument

$$b^e = n$$

base

argument　　　exponent

$$\log_b n = e$$

base

Note: This e is not the same e that represents the natural log.

Example 1:
Determine the value of y in each exponential equation.

(A) $81 = 10^y$

$\underset{b}{10}^{\overset{e}{y}} = \underset{n}{81}$

$\log_{10} 81 = y$

$y = \log_{10} 81$

$y = 1.908$

(B) $32 = 10^y$

$10^y = 32$

$\log_{10} 32 = y$

$y = 1.505$

Example 2:

(A) $45 = 10^y$

$\log_{10} 45 = y$

$y = 1.653$

(B) $25 = e^y$

$\log_e 25 = y$

$\ln 25 = y$ "lawn"

$y = 3.218$

Example 3:

Write the following logarithms in exponential form.

(A) $\log_{\underset{b}{2}} \overset{n}{8} = \overset{e}{3}$

$2^3 = 8$

(B) $\log_3 81 = 4$

$3^4 = 81$

(C) $\log_4 64 = 3$

$4^3 = 64$

Example 4:
Solve for y:

(A) $\log_{2} 16 = y$

b n e

$2^y = 16$

$2^y = 2^4$

$y = 4$

(B) $y = \log_4 1024$

$4^y = 1024$

$4^y = 4^5$

$y = 5$

Sometimes, the variable y might not be written in the logarithmic equation. In this case, we can put y in on our own and solve for it.

Example 5:
Solve each of the following.

(A) $\log_3\left(\frac{1}{27}\right) = y$

$3^y = \frac{1}{27}$

$3^y = \frac{1}{3^3}$

$3^y = 3^{-3}$

$y = -3$

(B) $\log_{\left(\frac{1}{4}\right)}(64) = y$

$\left(\frac{1}{4}\right)^y = 64$

$4^{-1\cdot y} = 4^3$

$4^{-y} = 4^3$

$-y = 3$

$y = -3$

(C) $\log_{64} 4 = y$

$64^y = 4$

$4^{3 \cdot y} = 4^1$

$3y = 1$

$y = \frac{1}{3}$

(D) $\log_5 \left(\frac{1}{25}\right) = y$

$5^y = \frac{1}{25}$

$5^y = \frac{1}{5^2}$

$5^y = 5^{-2}$

$y = -2$

(E) $\log_4(-4) = y$ can't take log of a negative number.

$4^y \neq -4$

Example 6:
Evaluate each expression:

(A) $\log_2 16 + \log_2 2$

$y = \log_2 16$

$2^y = 16$

$2^y = 2^4$

$y = 4$

$y = \log_2 2$

$2^y = 2$

$2^y = 2^1$

$y = 1$

$4 + 1 = 5$

or

$\log_2 16 + \log_2 2 = 5$

(B) $\log_2 1 - \log_2 \left(\frac{1}{8}\right)$

$y = \log_2 1$

$2^y = 1$

$y = 0$

$y = \log_2 \left(\frac{1}{8}\right)$

$2^y = \frac{1}{8}$

$2^y = \frac{1}{2^3}$

$2^y = 2^{-3}$

$y = -3$

$0 - (-3) = 3$

or

$\log_2 1 - \log_2 \left(\frac{1}{8}\right) = 3$

Special Cases - Base 10 or Base e
Whenever the base of a logarithm is 10, we can use the log button on our calculator to evaluate it. Whenever the base of a logarithm is e, we can use the **ln** button on our calculator to evaluate it.

Example 7:
Use a calculator to evaluate the following:

(A) $\log 100$

$= 2$ or $y = \log_{10} 100$

$10^y = 100$

$10^y = 10^2$

$y = 2$

(B) $\ln 20$

$= 2.996$

Evaluating Exponential Equations With it is NOT Possible to Get a Common Base (base 10 and *e* only)

Example 8: Write the following in logarithmic form.

(A) $81 = 10^y$

$y = \log_{10} 81$

$y = 1.9084$

(B) $25 = e^y$

$y = \ln 25$

$y = 3.2188$

Example 9:

(A) $45 = 10^y$

$y = \log 45 = 1.6532$

(B) $32 = e^y$

$y = \ln 32$

$y = 3.4657$

In Summary

Key Ideas

- The logarithmic function

$$y = \log_b x$$

is equivalent to the following exponential function:

$$x = b^y$$

- A logarithm is an exponent. The expression $\log_b x$ means "the exponent that must be applied to base b to give the value of x." For example:

$$\log_2 8 = 3$$

since

$$2^3 = 8$$

- The value of a logarithm can be determined in one of the following ways:
  - Set the logarithmic expression equal to y, and write the equivalent exponential form. Then determine the exponent to which the base must be raised to get the required number.
  - If the base of the logarithm is 10 or e, you can use a scientific or graphing calculator.

Need to Know

- The common logarithmic function
$$y = \log x$$
is equivalent to the following exponential function:
$$x = 10^y$$
- The natural logarithmic function
$$y = \ln x$$
is equivalent to the following exponential function:
$$x = e^y$$
- The logarithm of a negative number does not exist, because a negative number cannot be written as a power of a positive base.
- Many real-life situations have values that vary greatly. A logarithmic scale with powers of 10 can be used to make comparisons between large and small values more manageable. Three examples of logarithmic scales are the Richter scale (used to measure the magnitude of an earthquake), the pH scale (used to measure the acidity of a solution), and the decibel scale (used to measure sound level).

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