Math 3201

7.2B Problems Involving Logarithmic Scales

Case 1: pH of a Solution

The pH of a solution is determined using the equation $p(x) = -\log x$ where x is the concentration of hydrogen ions measured in moles per litre (mol/L). This scale ranges from 0 to 14 with the lower numbers being acidic and the higher numbers being basic. A value where the pH = 7 is considered neutral. The scale is a logarithmic scale with one unit of increase in pH resulting in a 10 fold decrease in acidity. Another way to consider this would be a one unit increase in pH results in a 10 fold increase in basicity.

		Examples of solutions
acidity increases in this direction	pH = 0	acid used in car batteries
	pH = 1	acid secreted by lining of stomach
	pH = 2	lemon juice, vinegar
	pH = 3	grapefruit, orange juice, pop
	pH = 4	tomato juice, acid rain
	pH = 5	soft drinking water, black coffee
	pH = 6	saliva
neutral	pH = 7	distilled water
	pH = 8	seawater
	pH = 9	baking soda
alkalinity increases in this direction	pH = 10	milk of magnesia
	pH = 11	ammonia solution used in cleaning products
	pH = 12	soapy water
	pH = 13	bleaches, oven cleaner
	pH = 14	chemical used in liquid drain cleaner

A neutral solution, such as distilled water, has a pH of 7. An acidic solution, such as pop, has a pH that is less than 7. The lower the pH, the more acidic the solution is. An alkaline solution, such as milk of magnesia, has a pH that is greater than 7. The higher the pH, the more alkaline the solution is.

An increase or decrease of one unit on the pH scale indicates a change in the hydrogen ion concentration by a factor of 10. For example, a solution with a pH of 4 has 10 times the hydrogen ion concentration of a solution with a pH of 5 and, as a result, is 10 times more acidic.

pH $\longrightarrow$ $p(x) = -\log x$ $\longleftarrow$ concentration of H^+ ions

Example 1:

(A) The hydrogen ion concentration, x, of a solution is 0.0001 mol/L. Calculate the pH of the solution.

$p(x) = -\log x$

$x = 0.0001$

$p(x) = -\log(0.0001)$

$pH = 4$

(B) Use the pH scale shown to calculate the hydrogen ion concentration of lemon juice.

$pH = 2 \leftarrow p(x)$

$2 = -\log X$

$\log X = -2$

$\log_{10} X = -2$

$10^{-2} = X$

$\frac{1}{100} = x$

$x = 0.01$ mol/L

(C) In terms of hydrogen ion concentration, how much more acidic is Solution A, with a pH of 1.6, than Solution B, with a pH of 2.5? Round your answer to the nearest 10th.

Solution A

$pH = 1.6$

$1.6 = -\log X$

$\log X = -1.6$

$10^{-1.6} = X$

$X = 0.02512$ mol/L

Solution B

$pH = 2.5$

$2.5 = -\log X$

$\log X = -2.5$

$10^{-2.5} = X$

$X = 0.003162$ mol/L

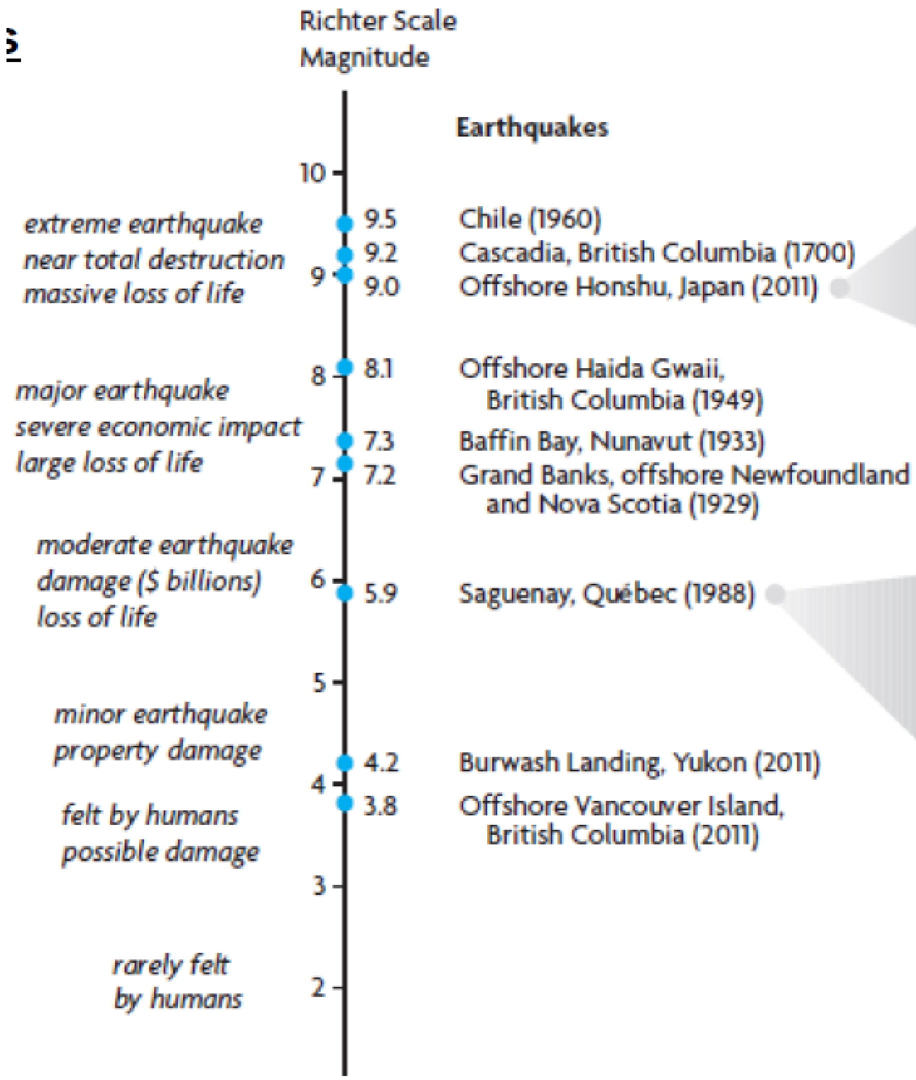
$\frac{\text{concentration A}}{\text{concentration B}} = \frac{0.02512}{0.003162} = 7.9$

A is 7.9 times more acidic than B.

Case 2: Magnitude of Earthquakes

The magnitude of an earthquake is measured on the Richter Scale. Magnitude refers to the amount of energy released during an earthquake.

The magnitude of an earthquake, y, can be determined using $y = \log x$, where x is the amplitude of the vibrations measured using a seismograph. An increase of one unit in magnitude results in a 10 fold increase in the amplitude. This topic lends itself to the incorporation of current events with the inclusion and comparison of a variety of earthquakes.



Example 2:
Determine the magnitude of an earthquake if the amplitude of vibrations, as measured by a seismograph, is 500.

$X = 500$

$Y = ?$

$Y = \log 500$

$Y = 2.7 \leftarrow$ magnitude

Case 3: Decibel Scale for Sound Levels

Example 3:
Sound levels are measured in decibels using the function $\beta = 10\ (\log I + 12)$, where β is the sound level in decibels (dB) and I is the sound intensity measured in watts per metre squared (W/m^2). What is the sound level, to the nearest decibel, of each sound?

(A) Rustle of leaves, if $I = 1 \times 10^{-11}\ W/m^2$

$\beta = 10\left[\log(1 \times 10^{-11}) + 12\right]$

$\beta = 10(-11 + 12)$

$\beta = 10(1)$

$\beta = 10\text{ dB}$

(B) Vacuum cleaner, if $I = 1 \times 10^{-4}\ W/m^2$

$\beta = 10\left[\log(1 \times 10^{-4}) + 12\right]$

$\beta = 10(-4 + 12)$

$\beta = 10(8)$

$\beta = 80\text{ dB}$

(C) Rock band, if $I = 0.01\ W/m^2$

$\beta = 10\left[\log(0.01) + 12\right]$

$\beta = 10(-2 + 12)$

$\beta = 10(10)$

$\beta = 100\text{ dB}$

(D) Jet Engine, if $I = 100\ W/m^2$

$\beta = 10\left[\log(100) + 12\right]$

$\beta = 10(2 + 12)$

$\beta = 10(14)$

$\beta = 140\text{ dB}$

Textbook Questions: page 437 #14 (a, b, c), 15 (a, b, c), 16(a), 18(b, c, d), 19, 20