**Math 3201**

# 7.3 Laws of Logarithms

Recall the Laws of Exponents from Math 9 and Math 1201:

Product: $a^m \times a^n = a^{m+n}$

Quotient: $a^m \div a^n = a^{m-n}$

Power of a Power: $(a^m)^n = a^{m \times n}$

Similarly, there are also Laws of Logarithms:

Product rule: $\log_b(m \times n) = \log_b m + \log_b(n)$

Quotient rule: $\log_b(m \div n) = \log_b m - \log_b(n)$

Power Law: $\log_b m^n = n\log_b m$

Change of Base Law: $log_b n = \frac{\log n}{\log b}$

These laws apply when.: $b > 0$, $m > 0$, $n > 0$ and $b \neq 1$, where $b$, $m$, and $n \in R$

**Note:** These laws also apply to natural logarithms, ln.

**Verification of Rules of Logarithms**

**Product rule:**

$$\log_3(9 \times 27) = \log_3 9 + \log_3 27$$

$y = \log_3(243)$
$3^y = 243$
$3^y = 3^5$
$y = 5$

$y = \log_3 9$
$3^y = 9$
$3^y = 3^2$
$y = 2$

$y = \log_3 27$
$3^y = 27$
$3^y = 3^3$
$y = 3$

$2 + 3$
$= 5$

**Quotient Rule:**

$$\log_2(256 \div 32) = \log_2 256 - \log_2 32$$

$Y = \log_2(8)$
$2^Y = 8$
$2^Y = 2^3$
$Y = 3$

$Y = \log_2 256$
$2^Y = 256$
$2^Y = 2^8$
$Y = 8$

$Y = \log_2 32$
$2^Y = 32$
$2^Y = 2^5$
$Y = 5$

$8 - 5$
$= 3$

**Power Law:**

$$^3\log_2 4^3 = 3\log_2 4$$

$Y = \log_2 4^3$
$Y = \log_2 64$
$2^Y = 64$
$2^Y = 2^6$
$Y = 6$

$3 \cdot (\log_2 4)$
$3(2)$
$= 6$

$Y = \log_2 4$
$2^Y = 4$
$2^Y = 2^2$
$Y = 2$

**Examples 1:**
Use the laws of logarithms to write the following as single logarithms, and then evaluate.

(A)

$$\log_2 5 + \log_2 6.4$$

$Y = \log_2 (5 \times 6.4)$

$Y = \log_2 (32)$ or $Y = \log_2 (32)$

$2^Y = 32$ $\qquad Y = \frac{\log 32}{\log 2}$

$2^Y = 2^5$

$Y = 5$ $\qquad Y = 5$

(B)

$$\log_5 100 - \log_5 4$$

$Y = \log_5 \left(\frac{100}{4}\right)$

$Y = \log_5 (25)$ or $Y = \log_5 25$

$5^Y = 25$ $\qquad Y = \frac{\log 25}{\log 5}$

$5^Y = 5^2$

$Y = 2$ $\qquad Y = 2$

(C)

$$\log_3 27^5$$

$= 5\log_3 27$

$= 5(y = \log_3 27)$ or $= 5\left[\dfrac{\log 27}{\log 3}\right]$

$= 5(3^y = 27)$

$= 5(3^y = 3^3)$

$= 5(3)$ $= 5(3)$

$= 15$ $= 15$

(D)

$$\log_3 18 + \log_3\left(\frac{3}{2}\right)$$

$y = \log_3\left(18 \cdot \frac{3}{2}\right)$

$y = \log_3 27$ or $y = \dfrac{\log 27}{\log 3}$

$3^y = 27$

$3^y = 3^3$

$y = 3$ $y = 3$

(E)

$$\log_5 40 - 3\log_5 2$$

$y = \log_5 40 - \log_5 2^3$

$y = \log_5 40 - \log_5 8$

$y = \log_5 \left(\frac{40}{8}\right)$

$y = \log_5 (5)$

$5^y = 5^1$

$y = 1$

**Example 2:**
Identify any errors and write the correct solution:

Simplify: $\log_5 36 + 2\log_5 3$

Response A: $\log_5 36 + \log_5 3^2$
$\log_5 36 + \log_5 6 \leftarrow 9$
$\log_5(36 \times 6)$
$\log_5 324$

Response B: $\log_5 36 + \log_5 3^2$
$\log_5 36 + \log_5 9$
$\log_5(36 \div 9)$ x
$\log_5 4$

$y = \log_5 36 + \log_5 3^2$

$y = \log_5 36 + \log_5 9$

$y = \log_5 (36 \cdot 9)$

$y = \log_5 324$

$y = \frac{\log 324}{\log 5}$

$y = 3.592$

**Example 3:**
Identify any errors and write the correct solution:

$$\frac{1}{2}\log_2 64-(2\log_2 6-\frac{1}{2}\log_2 81)$$

$$=\log_2 64^{\frac{1}{2}}-(\log_2 6^2-\log_2 81^{\frac{1}{2}})$$

$$=\log_2 8-(\log_2 12-\log_2 9)$$

36

$$=\log_2 8-(\log_2 3)$$

$\frac{12}{9}$

$$=\log_2 5$$

$=\log_2 64^{\frac{1}{2}}-(\log_2 6^2-\log_2 81^{\frac{1}{2}})$

$=\log_2\sqrt{64}-(\log_2 36-\log_2\sqrt{81})$

$=\log_2 8-(\log_2 36-\log_2 9)$

$=\log_2 8-\log_2\left(\frac{36}{9}\right)$

$=\log_2 8-\log_2 4$

$=\log_2\left(\frac{8}{4}\right)$

$=\log_2 2$

$=1$

$y=\log_2 2$

$2^y=2$

$y=1$

## In Summary

### Key Ideas

- The laws of logarithms are directly related to the exponent laws, since logarithms are exponents.
- The laws of logarithms can be used to simplify logarithmic expressions if all the terms have the same base.

### Need to Know

- The laws of logarithms can be expressed as follows, where $b, m, n > 0$ and $b \neq 1$, and $b$, $m$, and $n$ are real numbers:

  - **Product Law of Logarithms:** $\log_b mn = \log_b m + \log_b n$
    For example: $\log(5 \cdot 200) = \log 5 + \log 200$
  - **Quotient Law of Logarithms:** $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
    For example: $\ln\left(\frac{3}{4}\right) = \ln 3 - \ln 4$
  - **Power Law of Logarithms:** $\log_b m^n = n\log_b m$
    For example: $\log_2 5^3 = 3\log_2 5$

**Textbook Questions:** page 446 – 447 #1 - 7, 10, 11, 13, 14, 15, 16