Math 3201

7.4A Solving Exponential Equations Using Logarithms

Steps:

- Take the log (base 10) of each side of the equation.
- Use the power law to bring the exponents down.
- If the exponent has more than one term, use the distributive property.
- Group and combine like terms to solve the equation.

Example 1:
Solve for the variable.

(A)

$3^{x-1} = 20$

$\log 3^{(x-1)} = \log 20$

$(x-1)\log 3 = \log 20$

$\frac{(x-1)\log 3}{\log 3} = \frac{\log 20}{\log 3}$

$x - 1 = \frac{\log 20}{\log 3}$

$x - 1 = 2.7268$

$x = 2.7268 + 1$

$x = 3.7268$

(B)

$2^x = 3^x$

$\log 2^x = \log 3^x$

$x\log 2 = x\log 3$

$x\log 2 - x\log 3 = 0$

$x(\log 2 - \log 3) = 0$

$\frac{x(\log 2 - \log 3)}{(\log 2 - \log 3)} = \frac{0}{\log 2 - \log 3}$

$x = 0$

(C) ~~Public~~

$2^{x-1} = 3^{x+1}$

$\log 2^{(x-1)} = \log 3^{(x+1)}$

$(x-1)\log 2 = (x+1)\log 3$

$x\log 2 - \log 2 = x\log 3 + \log 3$

$x\log 2 - x\log 3 = \log 3 + \log 2$

$x(\log 2 - \log 3) = \log 3 + \log 2$

$\frac{-0.1761x}{-0.1761} = \frac{0.7782}{-0.1761}$

$x = -4.4$

(D)

$4(3^{2x}) = 24$

$\frac{4(3^{2x})}{4} = \frac{24}{4}$

$3^{2x} = 6$

$\log 3^{2x} = \log 6$

$\frac{2x\log 3}{\log 3} = \frac{\log 6}{\log 3}$

$\frac{2x}{2} = \frac{1.6309}{2}$

$x = 0.8155$

or

$\log_3 3^{2x} = \log_3 6$

$2x = \log_3 6$

$\frac{2x}{2} = \frac{\log 6}{\log 3} \div 2$

$x = 0.8155$

Solving Log Equations Using a Calculator

Recall the examples we did when we evaluated a logarithm by changing to exponential form:

$\log_3 9 = y$

$3^y = 9$

$3^y = 3^2$

$y = 2$

or

$y = \log_3 9$

$y = \frac{\log 9}{\log 3}$

$y = 2$

This strategy works fine when we can get a common base, but is not useful in cases in which we cannot get a common base. In these cases however, we can evaluate the logarithm using a calculator. The **change of base rule** has already been introduced. This rule is **not** provided on the final exam.

$$\log_b n = \frac{\log n}{\log b}$$

Example 2:

(A)

$y = \log_2 9$

$y = \frac{\log 9}{\log 2}$

$y = 3.1699$

(B)

$y = \log_{\frac{1}{2}} 0.1$

$y = \frac{\log 0.1}{\log \frac{1}{2}}$

$y = 3.322$

Textbook Questions: page 455 - 457 #1, 2, 3, 5, 6, 15, 16