**Math 3201**

# 7.4B Word Problems Involving Logarithms

Here will we look at application problems very similar to those back in Unit 6, except this time we will have to use logarithms to solve them.

**Interest Problems**

Recall the following from Unit 6 about compound interest problems:

$$A = P(1+i)^n$$

Where :

- $P$ is the principle amount
- $i$ is the interest rate per compounding period
- $n$ is the number of compounding periods

Notice that $i$ is the interest rate per compounding period.

Compounding periods are usually daily, weekly, semimonthly, monthly, quarterly, semiannually or annually. The table below shows how many times interest is paid, and the interest rate for each of these options.

| Compounding Period | Number of Times Interest Is Paid | Interest Rate per Compounding Period, $i$ |
|---|---|---|
| daily | 365 times per year | $i = \frac{\text{annual rate}}{365}$ |
| weekly | 52 times per year | $i = \frac{\text{annual rate}}{52}$ |
| semi-monthly | 24 times per year | $i = \frac{\text{annual rate}}{24}$ |
| monthly | 12 times per year | $i = \frac{\text{annual rate}}{12}$ |
| quarterly | 4 times per year | $i = \frac{\text{annual rate}}{4}$ |
| semi-annually | 2 times per year | $i = \frac{\text{annual rate}}{2}$ |
| annually | 1 time per year | $i = \frac{\text{annual rate}}{1}$ |

**Example 1**:
If a $100 deposit is made at a bank that pays 12% per year, compounded annually, determine how long it will take for the investment to reach $2000.

$P = 100$

$A = 2000$

$i = \dfrac{0.12}{1} = 0.12$

$n = ?$

$A = P(1+i)^n$

$2000 = 100(1+0.12)^n$

$\dfrac{2000}{100} = \dfrac{100(1.12)^n}{100}$

$20 = 1.12^n$

$\log 20 = \log 1.12^n$

$\dfrac{\log 20}{\log 1.12} = \dfrac{n \cancel{\log 1.12}}{\cancel{\log 1.12}}$

$26.4 = n$

<u>or</u>

${}_{n}20 = \underset{b}{1.12}^{\overset{e}{n}}$

$\log_{1.12} 20 = n$

$\dfrac{\log 20}{\log 1.12} = n$

$26.4 = n$

It will take 26.4 years for $100 investment to be worth $2000.

**Example 2:**

Kelly invests \$5000 with a bank. The value of her investment can be determined using the formula $y = 5000(1.06)^t$, where $y$ is the value of the invesetment at time $t$, in years. Approximately how many years will it take for Kelly's investment to reach \$20 000?

$$\frac{20000}{5000} = \frac{5000(1.06)^t}{5000}$$

$$\log 4 = \log 1.06^t \qquad 4 = 1.06^t$$

$$\frac{\log 4}{\log 1.06} = \frac{t \cancel{\log 1.06}}{\cancel{\log 1.06}}$$

$$t = 23.8 \text{ years}$$

$$\underset{n}{4} = \underset{b}{1.06}^{\,t^{e}}$$

$$\log_{1.06} 4 = t$$

$$\frac{\log 4}{\log 1.06} = t$$

$$t = 23.8 \text{ yers}$$

**Example 3:**
\$2000 is invested at 5% per year, compounded semi-annually. How long in months, will it take for the investment to triple in value?

$P = 2000$

$A = 3(2000) = 6000$

$i = \frac{0.05}{2} = 0.025$

$n = ?$

$6000 = 2000(1+0.025)^n$

$\frac{6000}{2000} = \frac{\cancel{2000}(1.025)^n}{\cancel{2000}}$

$3 = 1.025^n$

$\log 3 = \log 1.025^n$

$\frac{\log 3}{\log 1.025} = \frac{n \cancel{\log 1.025}}{\cancel{\log 1.025}}$

$n = 44.5$

time: $\frac{44.5}{2} = 22.25$ years.

$22.25 \cancel{\text{years}} \times \frac{12 \text{ months}}{1 \cancel{\text{year}}} = 267 \text{ months}$

**Example 4:**

The equation $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{3}}$ represents the radioactive sample where the half-life is 3 years. If the initial mass of the sample is 67g, how long will it take for the sample to reach 7g?

$A = 7$

$A_0 = 67$

$$\frac{7}{67} = \frac{67\left(\frac{1}{2}\right)^{\frac{t}{3}}}{67}$$

$$0.1045 = 0.5^{\frac{t}{3}}$$

$$\log 0.1045 = \log 0.5^{\frac{t}{3}}$$

$$\frac{\log 0.1045}{\log 0.5} = \frac{\frac{t}{3}\log 0.5}{\log 0.5}$$

$$3(3.2584) = \frac{t}{\cancel{3}} \cdot \cancel{3}$$

$t = 9.8$ years

**Example 5:**

The equation $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{15}}$ represents the population of a town that halves every 15 years, where $A$ represents the population remaining after a certain time. In how many years will the population decrease by 25%?

$A = 75\%$ or $0.75$

$A_0 = 100\%$ or $1$

$$\frac{75}{100} = \frac{100\left(\frac{1}{2}\right)^{\frac{t}{15}}}{100}$$

$$0.75 = 0.5^{\frac{t}{15}}$$

$$\log 0.75 = \log 0.5^{\frac{t}{15}}$$

$$\frac{\log 0.75}{\log 0.5} = \frac{\frac{t}{15}\cancel{\log 0.5}}{\cancel{\log 0.5}}$$

$$\cancel{15}\cdot\frac{t}{\cancel{15}} = 0.4150\cdot 15$$

$t = 6.2$ years

**Example 6:**
After taking a cough suppressant, the amount, $A$, in mg, remaining in the body is given by $A = 10(0.85)^t$, where $t$ is given in hours.

(A) What was initial amount taken?

10 mg

(B) What percent of the drug leaves the body each hour?

0.85 or 85%

(C) How much of the drug leaves the body 6 hours after the dose is administered?

$t = 6$

$A = ?$

$A = 10(0.85)^6$

$A = 3.77$ mg

(D) How long is it until only 1 mg of the drug remains in the body?

$$\frac{1}{10} = \frac{10(0.85)^t}{10}$$

$$0.10 = 0.85^t$$

$$\log 0.10 = \log 0.85^t$$

$$\frac{\log 0.10}{\log 0.85} = \frac{t \cancel{\log 0.85}}{\cancel{\log 0.85}}$$

$$t = 14.2\text{ h}$$

**Textbook Questions:** page 457 #8, 10, 11, 12