Math 3201

8.1 Understanding Angles

Measuring Angles in Degrees and Radians

Many quantities can be measured using different units. What are some different units used for length?

m, ft, cm, miles, km

What are some different units used for temperature?

°C, °F, K

What are some different units used for angle measures?

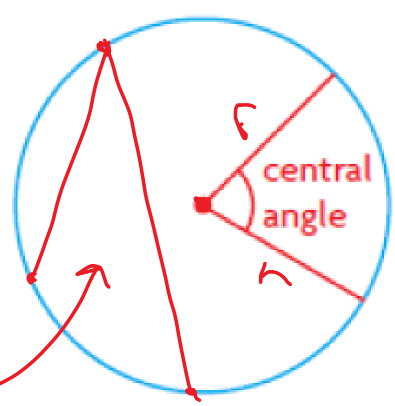
degrees, radians

Measuring Angles

Up to this point, we have only measured angles in the unit of degrees, for example 60°. Here we will learn about a different unit for measuring angles called **radians**. Before we learn about the unit of radians, there are some terms we need to look at.

Central Angle: an angle whose vertex is at the center of a circle.

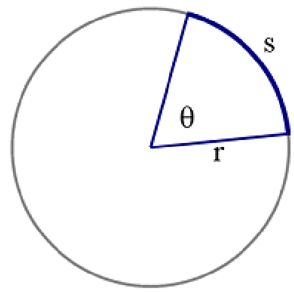
not a central angle



Unit Circle: a circle whose radius is 1.

Arc Length: a part of the circumference of a circle, s, subtended by a central angle.

θ = theta



The angle measure θ in radians equals the arc length s divided by the radius r. Since radian measure depends on arc length and radius, it is a good unit for us to use when graphing.

When we go on to graph sinusoidal functions, we will graph the amplitude or height of an object against an angle. If the angle is measured in radians instead of degrees, we will have consistent units on both the x and y axis of our graph and the graph will take on its true shape. If we used degrees as the unit for angle measures, the graph would be vertically exaggerated.

Investigating Radians

Answer the following:

1. Write an expression for the circumference of a unit circle $r = 1$.

$C = 2\pi r$

$C = 2\pi(1) = 2\pi$

2. How many degrees are in a full circle?

$360°$

If we equate the circumference and a full rotation around the unit we get:

$$2\pi \text{ radians} = 360^{\circ}$$

We will use this statement to help us convert back and forth between radians and degrees. Angles measured in radians can be written in decimal form, or they can be written as some multiple of π such as $2\pi, \frac{\pi}{4}$ and so on. Also, **angles in radians don't normally have a unit written after them.**

Conversion Factors

Changing Degrees to Radians

$$\theta \times \frac{\pi}{180^{\circ}} = \#\text{ radians}$$

Note: we want the answer in radians to have the π symbol on top, so it must be on top in the conversion factor.

Changing Radians to Degrees

$$\#\text{ radians} \times \frac{180^{\circ}}{\pi} = \text{degrees}$$

Note: we want the π symbol in the radian measure to cancel out for our degree value, so we must put it in the bottom of the conversion factor to ensure that it will cancel.

Example 1:
Convert the following angles from degrees to radians: $\times \frac{\pi}{180}$

(A) 90°

$$90^\circ \times \frac{\pi}{180^\circ} = \frac{90^\circ \pi}{180^\circ} = \frac{\pi}{2}$$

(B) 130°

$$130^\circ \times \frac{\pi}{180^\circ} = \frac{130^\circ \pi}{180^\circ} = \frac{13\pi}{18}$$

(C) 225°

$$225^\circ \times \frac{\pi}{180^\circ} = \frac{225^\circ \pi}{180^\circ} = \frac{5\pi}{4}$$

(D) 67°

$$67^\circ \times \frac{\pi}{180^\circ} = \frac{67\pi}{180}$$

(E) 324°

$$324^\circ \times \frac{\pi}{180^\circ} = \frac{9\pi}{5}$$

Example 2:
Convert the following angles from radians to degrees: $\times \frac{180^\circ}{\pi}$

(A) 4

$$4 \times \frac{180^\circ}{\pi} = \frac{720^\circ}{\pi} = 229.2^\circ$$

(B) 3.2

$$3.2 \times \frac{180^\circ}{\pi} = \frac{576^\circ}{\pi} = 183.3^\circ$$

(C) $\frac{\pi}{4}$

$$\frac{\cancel{\pi}}{4} \times \frac{180^\circ}{\cancel{\pi}} = \frac{180^\circ}{4} = 45^\circ$$

(D) $\frac{2\pi}{3}$

$$\frac{2\pi}{3} \times \frac{180^\circ}{\pi} = \frac{360^\circ}{3} = 120^\circ$$

Example 3:
How many degrees are in one radian?

$$1 \times \frac{180^\circ}{\pi} = 57.3^\circ$$

Example 4:
Which is larger? 3π or 8

$$3\cancel{\pi} \times \frac{180^\circ}{\cancel{\pi}} = 3 \cdot 180^\circ = 540^\circ$$

$$8 \times \frac{180^\circ}{\pi} = \frac{1440^\circ}{\pi} = 458^\circ$$

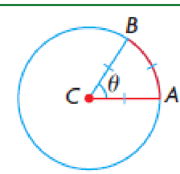
In Summary

Key Ideas

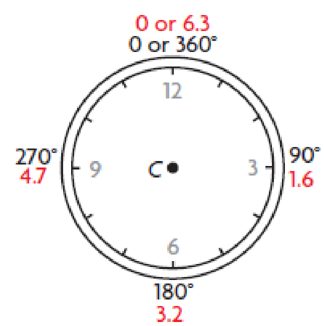
- Radian measure is an alternative way to express the size of an angle.
- Using radians allows you to express the measure of an angle as a real number without units.
- The central angle formed by one complete revolution in a circle is 360°, or 2π in radian measure.

Need to Know

- Use benchmarks to estimate the degree measure of an angle given in radians.
- In radian measure,
  - 1 is equivalent to about 60°;
  - π is equivalent to 180°;
  - 2π is equivalent to 360°.
- Decimal approximations can be used for benchmarks to visualize the approximate size of an angle measured in radians.



$\angle\theta = 57.3°$ or
1 in radian measure



Some estimation benchmarks

Textbook Questions: page 489 - 490 #1, 2, 4, 5, 8