

## 8.2 Exploring Graphs of Periodic Functions

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### Graphs of Sinusoidal Functions

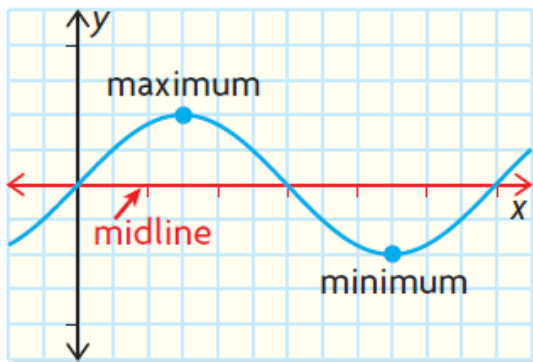
Many cycles/patterns exist in the world around us such as those in fabric prints, flooring and computer graphic designs.

When working with functions, if a pattern repeats regularly over some interval of the domain, then the function is **periodic**. A ferris wheel demonstrates periodic behavior since it completes one rotation every  $t$  minutes.

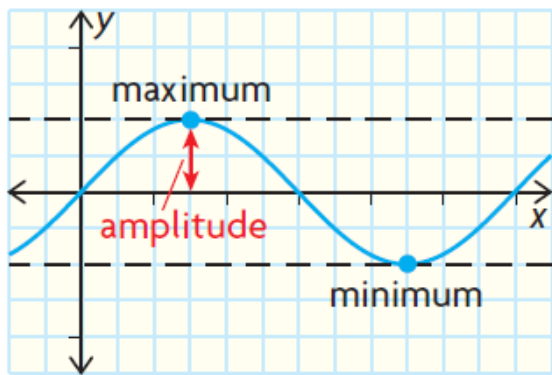
If we graph the sine and cosine functions, they will display periodic behavior. Before we start graphing sinusoidal functions, there is some terminology that we must be familiar with.

**Periodic Function:** a function whose graph repeats in a regular intervals or cycles.

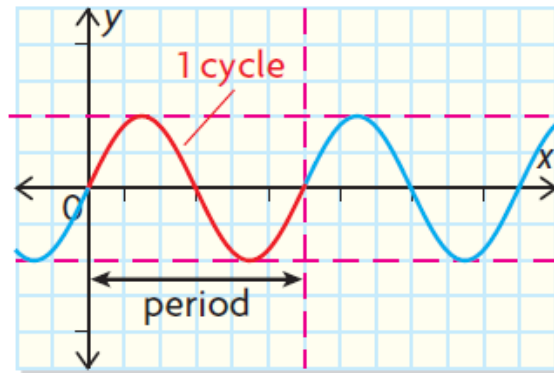
**Midline:** the horizontal line halfway between the maximum and the minimum values of a periodic function.



**Amplitude:** the distance from the midline to either the maximum or minimum of a periodic function; the amplitude is always expressed as a positive number.



**Period:** the length of the interval of the domain to complete one cycle.

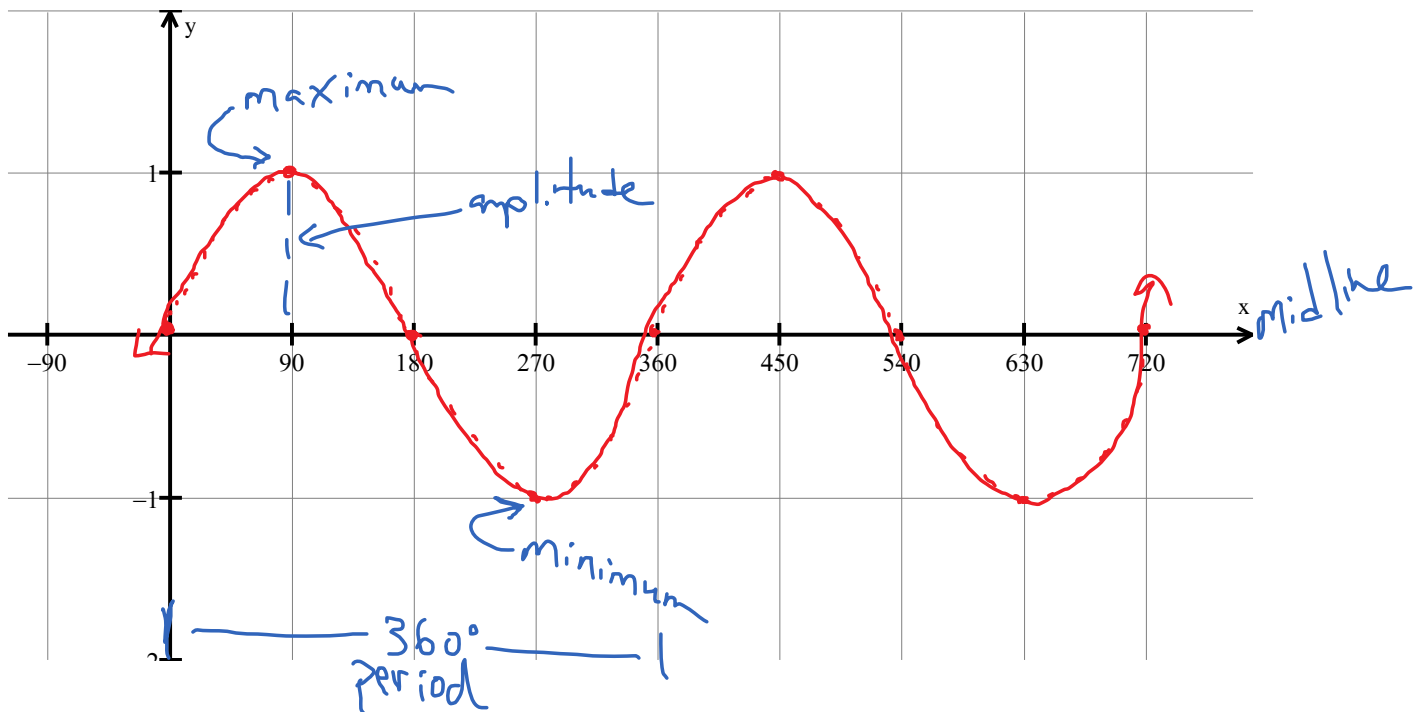


### The Sine Function

Complete the following table of values for  $y = \sin x$ .

$x$	$0^\circ$	$90^\circ$	$180^\circ$	$270^\circ$	$360^\circ$	$450^\circ$	$540^\circ$	$630^\circ$	$720^\circ$
$y$	0	1	0	-1	0	1	0	-1	0

Graph the data on the grid provided:



### Questions $y = \sin x$

1. What is the y-intercept or starting point?

$$y=0 \quad (0^\circ, 0)$$

2. What are the x-intercepts?

$$0^\circ, 180^\circ, 360^\circ, 540^\circ, 720^\circ$$

3. What is the maximum value?

$$y=1$$

4. What is the minimum value?

$$y=-1$$

5. At what values of  $\theta$  are there maximum values?

$$90^\circ, 450^\circ$$

6. At what values of  $\theta$  are there minimum values?

$$270^\circ, 630^\circ$$

7. Is the graph periodic?

yes. (repeats)

8. What is the period of the graph?

$$360^\circ$$

9. What is the amplitude of the graph?

$$1$$

10. What is the equation of the midline?

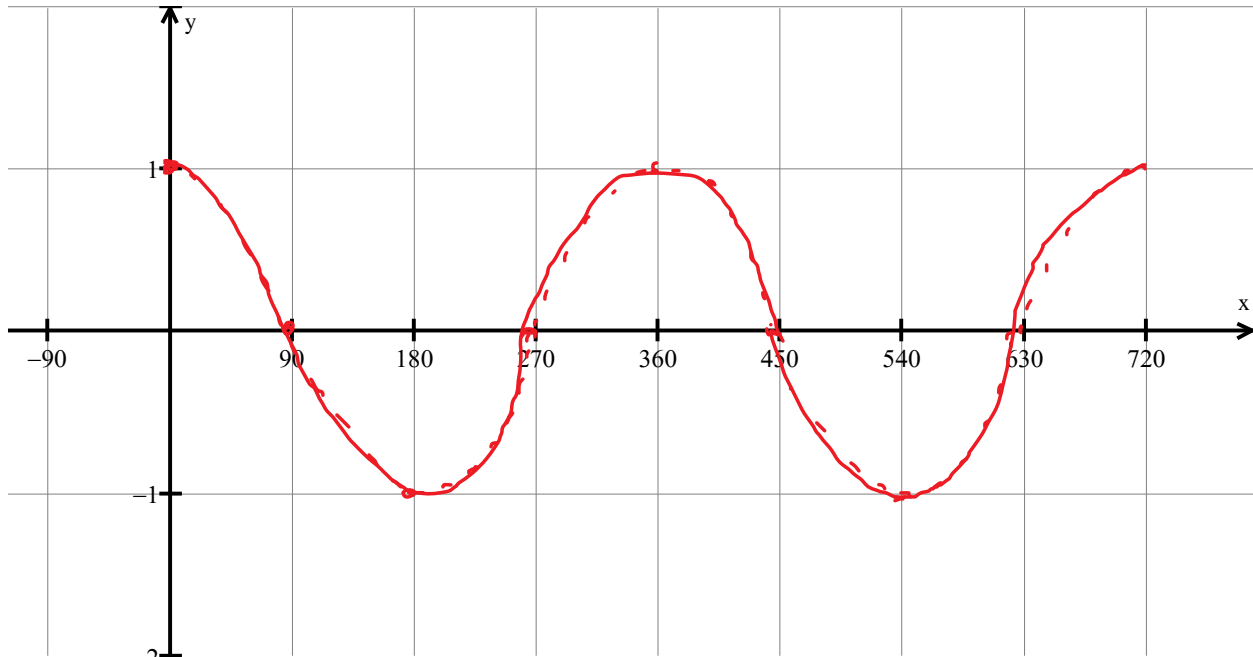
$$y=0$$

## The Cosine Function

Complete the following table of values for  $y = \cos x$ .

$x$	$0^\circ$	$90^\circ$	$180^\circ$	$270^\circ$	$360^\circ$	$450^\circ$	$540^\circ$	$630^\circ$	$720^\circ$
$y$	1	0	-1	0	1	0	-1	0	1

Graph the data on the grid provided:



Questions  $y = \cos x$ :

1. What is the y-intercept or starting point?

$$y = 1 \quad (0^\circ, 1)$$

2. What are the x-intercepts?

$$90^\circ, 270^\circ, 450^\circ, 630^\circ$$

3. What is the maximum value?

$$y = 1$$

4. What is the minimum value?

$$y = -1$$

5. At what values of  $\theta$  are there maximum values?

$$0^\circ, 360^\circ, 720^\circ$$

6. At what values of  $\theta$  are there minimum values?

$$180^\circ, 540^\circ$$

7. Is the graph periodic?

yes.

8. What is the period of the graph?

$$360^\circ$$

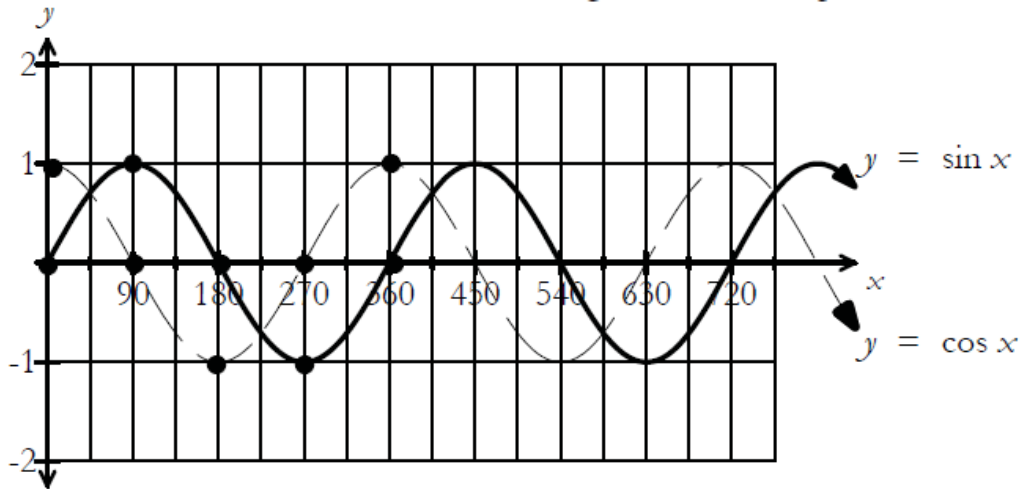
9. What is the amplitude of the graph?

$$1$$

10. What is the equation of the midline?

$$y = 0$$

## Comparing the Sine and Cosine Functions



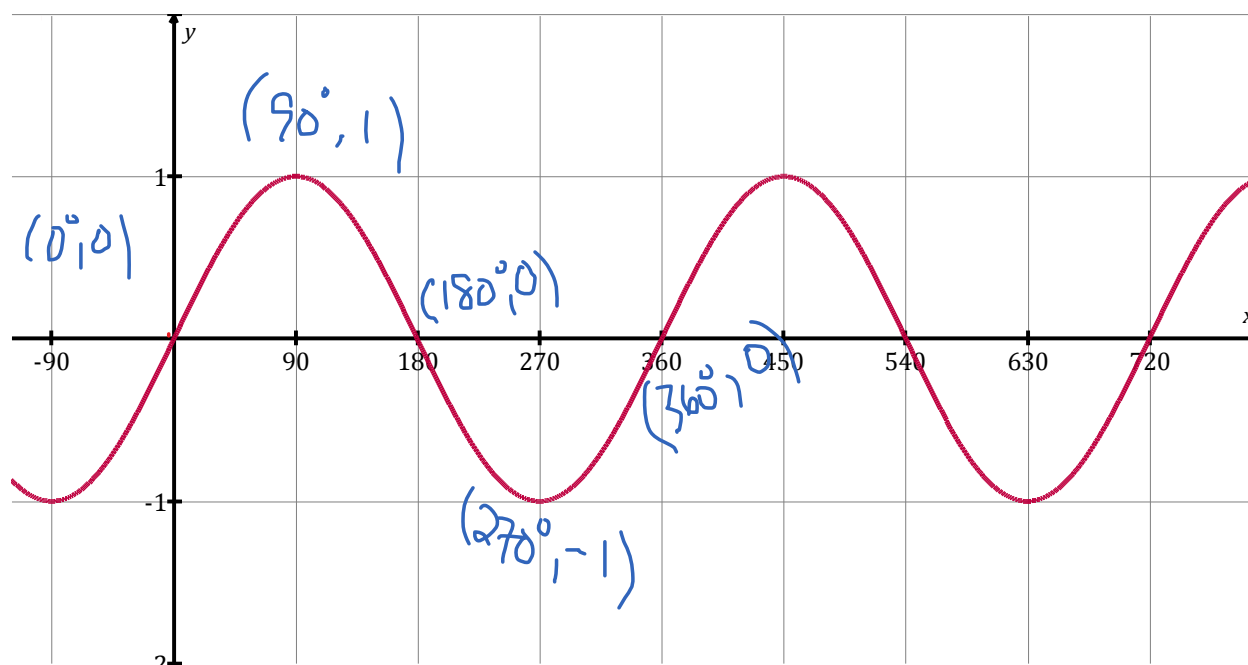
How are the graphs of the sine and cosine functions similar?

Same period, amplitude, maximum, minimum, midline.

How are the graphs of the sine and cosine functions different?

Cosine is essentially the sine graph shifted horizontally to the left (negative) by  $90^\circ$ .

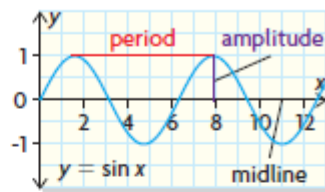
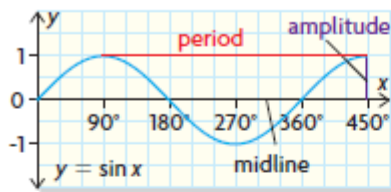
Look specifically at the graph of  $y = \sin x$ . Identify 5 key points that could enable us to sketch the graph even if we didn't know anything else other than these 5 points.



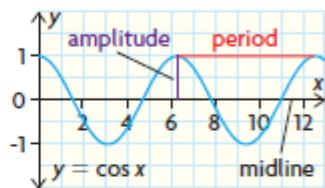
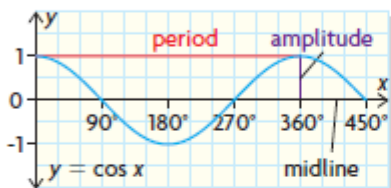
## In Summary

### Key Ideas

- The function  $y = \sin x$  is a periodic function.



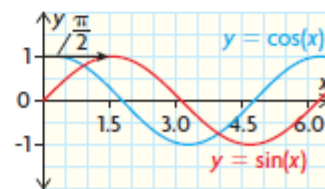
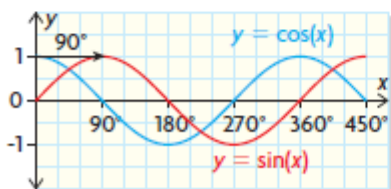
- The function  $y = \cos x$  is a periodic function.



- The graphs of  $y = \sin x$  and  $y = \cos x$  have the following common characteristics:
  - multiple  $x$ -intercepts
  - one  $y$ -intercept
  - a domain of  $\{x \mid x \in \mathbb{R}\}$
  - a range of  $\{y \mid -1 \leq y \leq 1, y \in \mathbb{R}\}$
  - an amplitude of 1
  - a period of  $360^\circ$  or  $2\pi$
  - a midline defined by the equation  $y = 0$

### Need to Know

- The graphs of  $y = \sin x$  and  $y = \cos x$  are congruent curves.



- The midline of the curves,  $y = 0$ , is the horizontal line halfway between the maximum and minimum values. The two graphs oscillate about this line.
- The period of a graph is the length of one complete cycle.