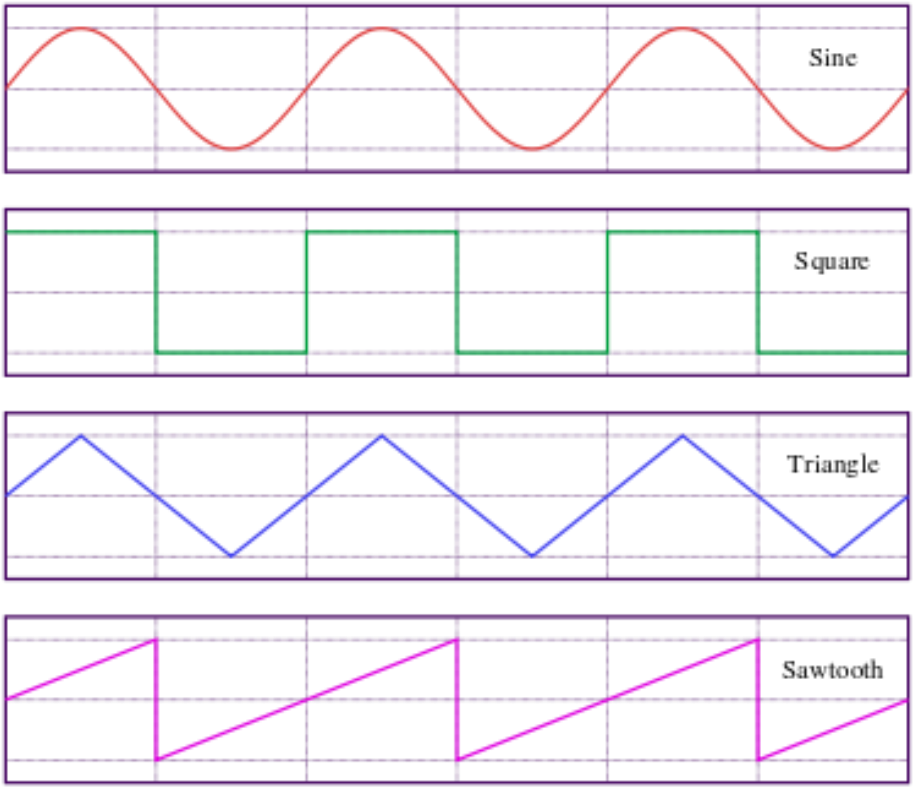
Math 3201

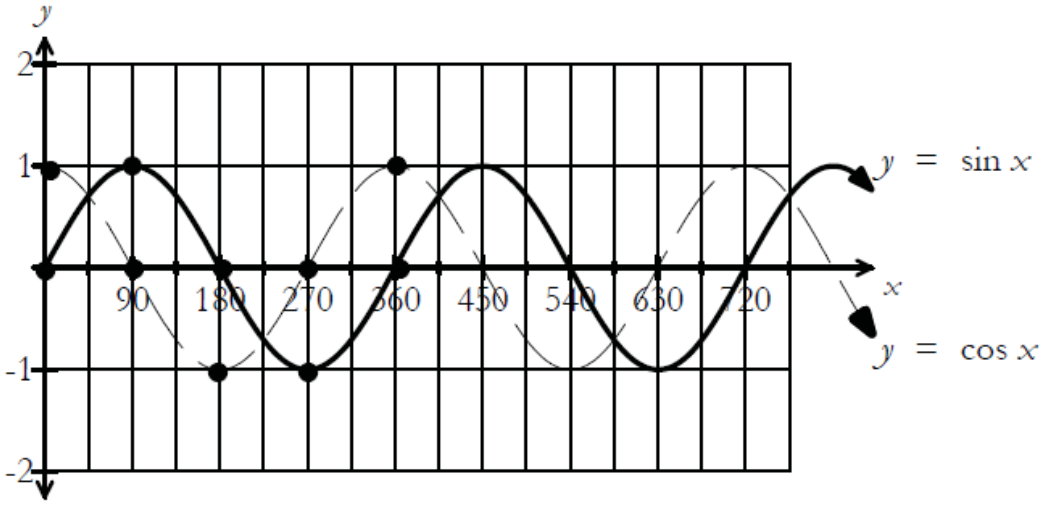
8.3 The Graphs of Sinusoidal Functions

Recall that a **periodic function** is one whose graph shows a repeating pattern.

Below are some examples of periodic graphs.



Only one of these graphs is sinusoidal, however. A **sinusoidal function** is one whose graph has the same shape as the graph of $y = \sin x$ and $y = \cos x$.



Example 1:

(A) Are all periodic graphs sinusoidal?

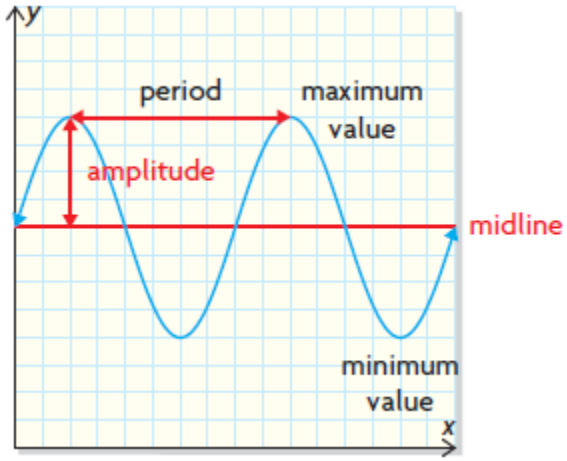
No. Some graphs are periodic but not sinusoidal.

(B) Are all sinusoidal graphs also periodic?

Yes. Sinusoidal graphs are periodic, smooth, wave-like.

Properties of Sinusoidal Graphs

Recall the terms midline, amplitude, period, minimum value and maximum value.



The maximum and minimum points can be easily read from a graph. The period, amplitude and the equation of the midline require a little more work to determine.

Period: the horizontal distance between consecutive maximum values or consecutive minimum values.

Equation of the Midline: the average of the maximum and minimum values found by the equation:

$$y = \frac{\text{maximum value} + \text{minumum value}}{2}$$

Amplitude: the positive vertical distance between the midline and the maximum value or the minimum value. It's also half the vertical distance between a maximum and a minimum value.

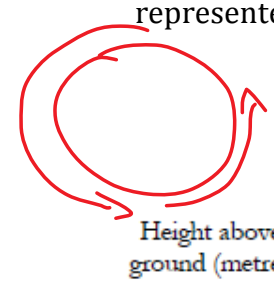
$$\text{Amplitude} = \frac{\text{maximum value} - \text{minumum value}}{2}$$

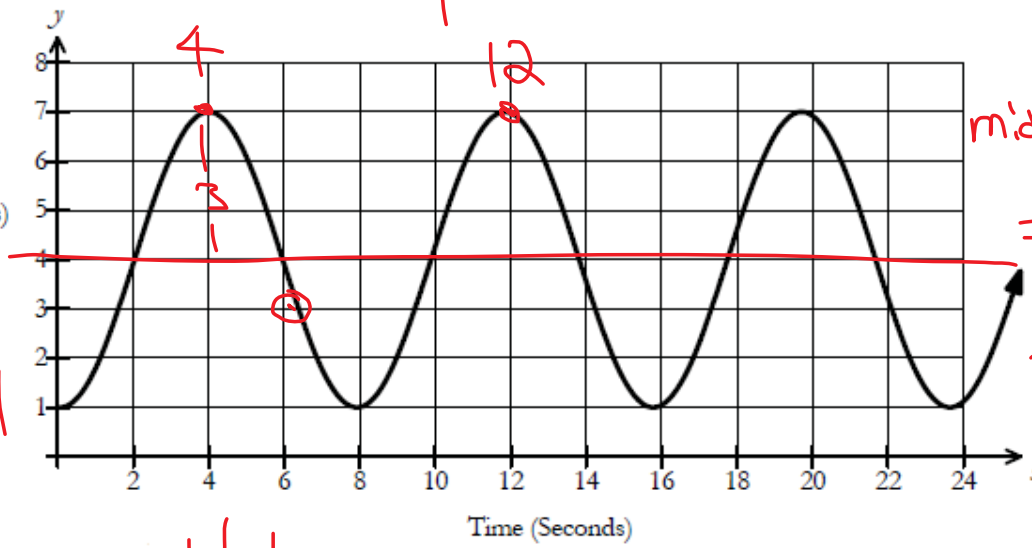
Range: the distance between the maximum and minimum values.

$$\text{Range} = \text{maximum value} - \text{minimum value}$$

Interpreting Sinusoidal Graphs

Example 2:
While riding a ferris wheel, Mason's height above the ground in terms of time can be represented by the following graph:





period = 12 − 4 = 8

4

12

middle = $\frac{\text{max} + \text{min}}{2}$ = $\frac{7+1}{2}$ = $\frac{8}{2}$ = 4

Range: 7 − 1 = 6

amplitude = $\frac{\text{max} - \text{min}}{2}$ = $\frac{7-1}{2}$ = $\frac{6}{2}$ = 3

(A) Identify the period of the graph.

$12-4=8s$

(B) Identify the range of the data.

range = max − min = 7 − 1 = 6

(C) Determine the equation of the midline.

$\frac{max+min}{2} = \frac{7+1}{2} = \frac{8}{2} = 4 \rightarrow y = 4$

(D) Identify the amplitude.

$\frac{max-min}{2} = \frac{7-1}{2} = \frac{6}{2} = 3$

(E) State the x and y intercepts.

x-int: none y-int: (0,1) or $y=1$

(F) What was Mason's height at 10 s?

4m (read from graph)

(G) At what time did Mason first reach a height of 7m?

4s (read from graph)

(H) What was the first time at which Mason reached a height of 3m while on the way down after reaching the maximum height?

6.5s.

Textbook Questions: page 506 - 511 #1, 2, 3, 4, 5, 8, 9, 13, 14