Math 3201

8.4B – 8.5 The Equations of Sinusoidal Functions

Recap from 8.4A

2 sinusoidal functions

(1) $y = a\sin b(x-c)+d$

(2) $y = a\cos b(x-c)+d$

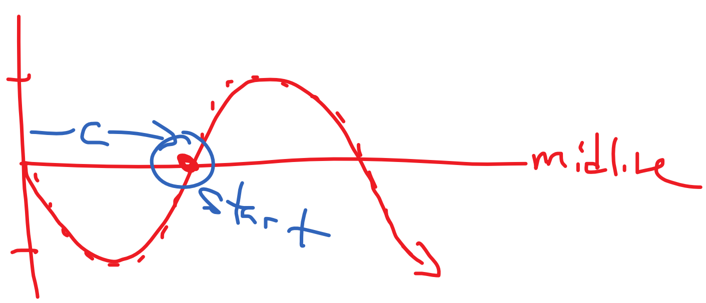
a: amplitude

b: period $= \frac{360°}{b}$

d: midline $y=d$

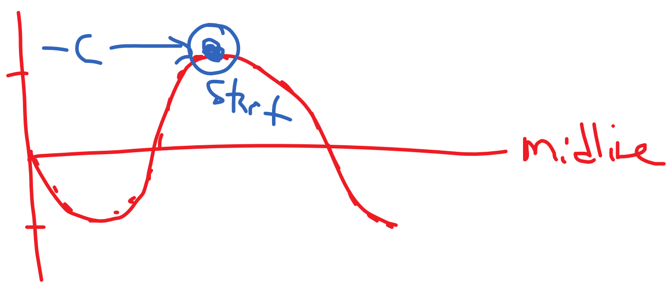
c:

For sin



c = x-value of the first point on the midline where graph is moving up.

For cosine



c = x-value of first max point.

Matching Sinusoidal Equations with Graphs

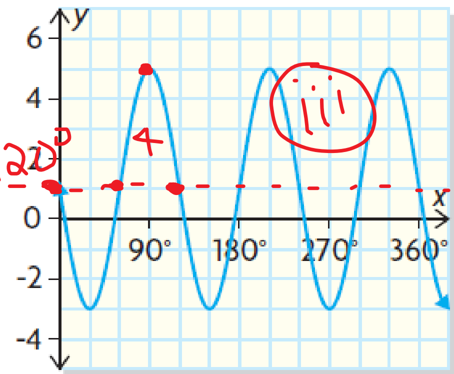
Example 1:
Match the following equations with the corresponding graphs.

midline $= \frac{5+(-3)}{2} = 1$

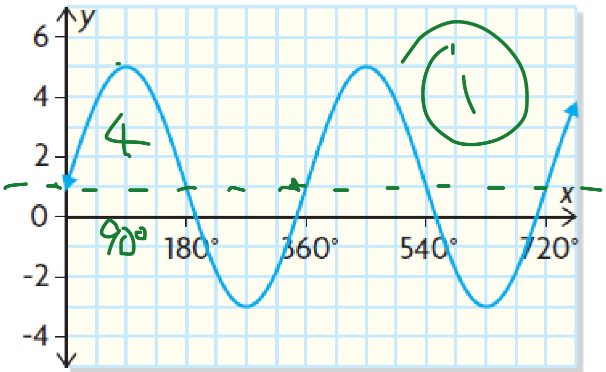
a) Graph 1 $d = 1$

$a = 4$

period $= 120°$



b) Graph 2



i) ~~$y = 4\cos(x - 90°) + 1$~~

ii) ~~$y = 5\sin 3(x - 60°)$~~

iii) $y = 4\sin 3(x - 60°) + 1$

iv) ~~$y = 4\cos 3(x - 60°) + 1$~~

$b = \frac{360°}{360°} = 1$

period $= \frac{360°}{b}$

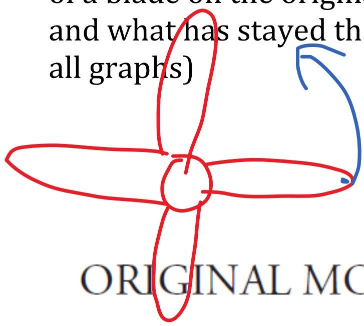
$b = \frac{360°}{\text{period}}$

$b = \frac{360°}{120°} = 3$

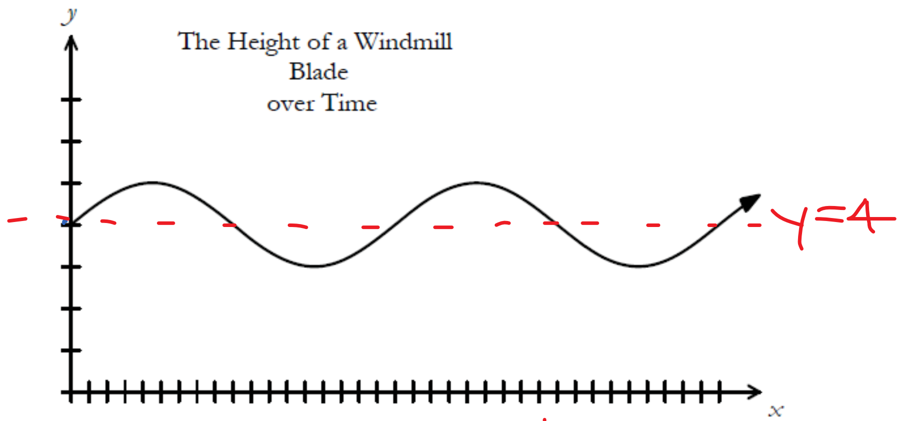
Textbook Questions: page 529 #13, 14

Applications of Sinusoidal Functions

Example 2:
A company is experimenting with a new type of windmill. The graph below shows the path of a blade on the original windmill over time. Ask students to describe what has changed and what has stayed the same in the new models. (Note: The scale is the same on all graphs)

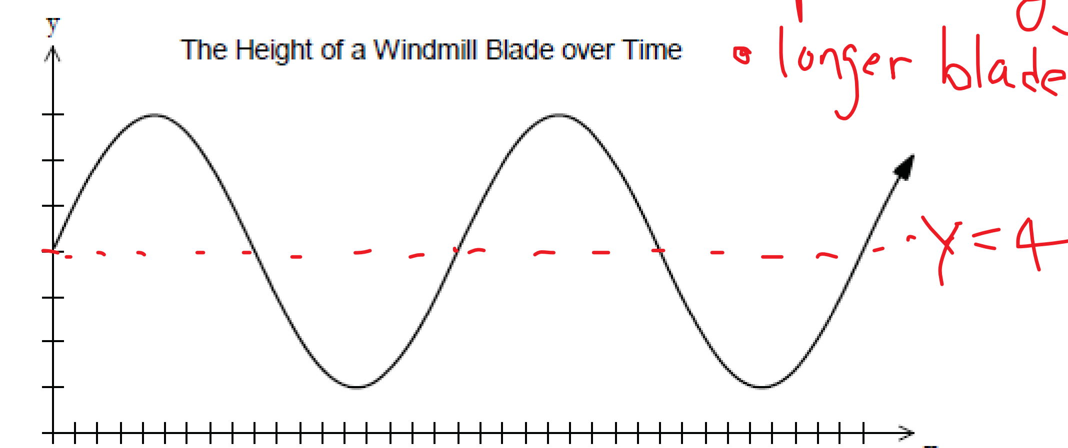


ORIGINAL MODEL



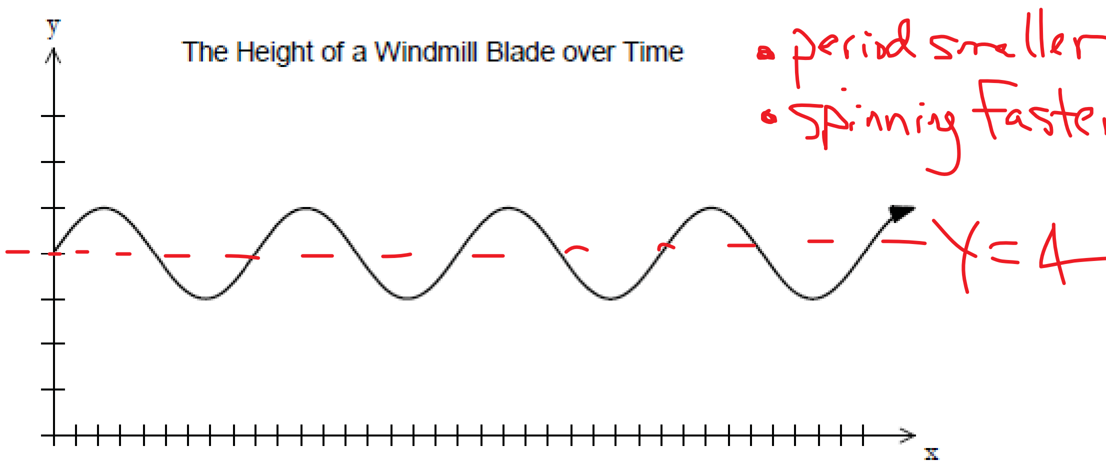
(A)

- amplitude bigger
- longer blades



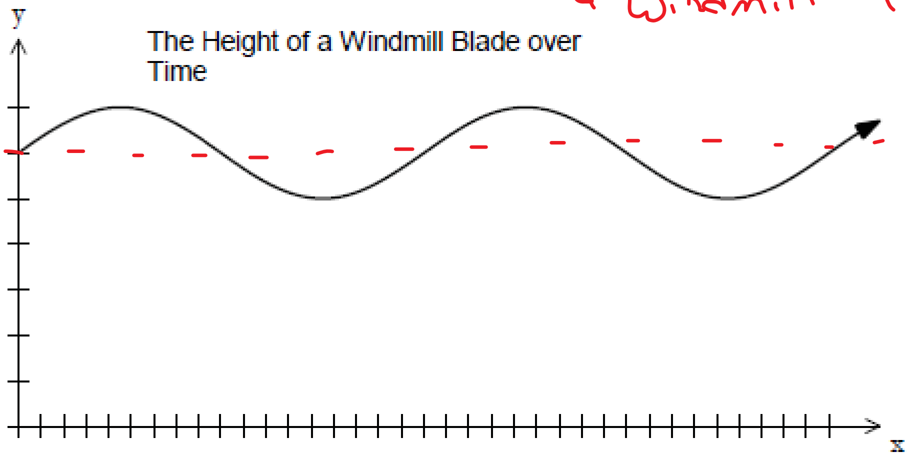
(B)

- period smaller
- spinning faster



(C)

• midline higher
• windmill taller



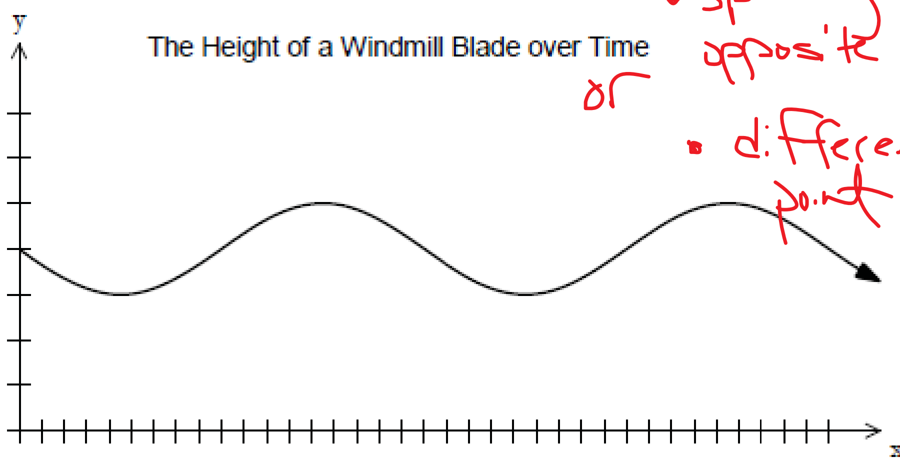
y = 6

(D)

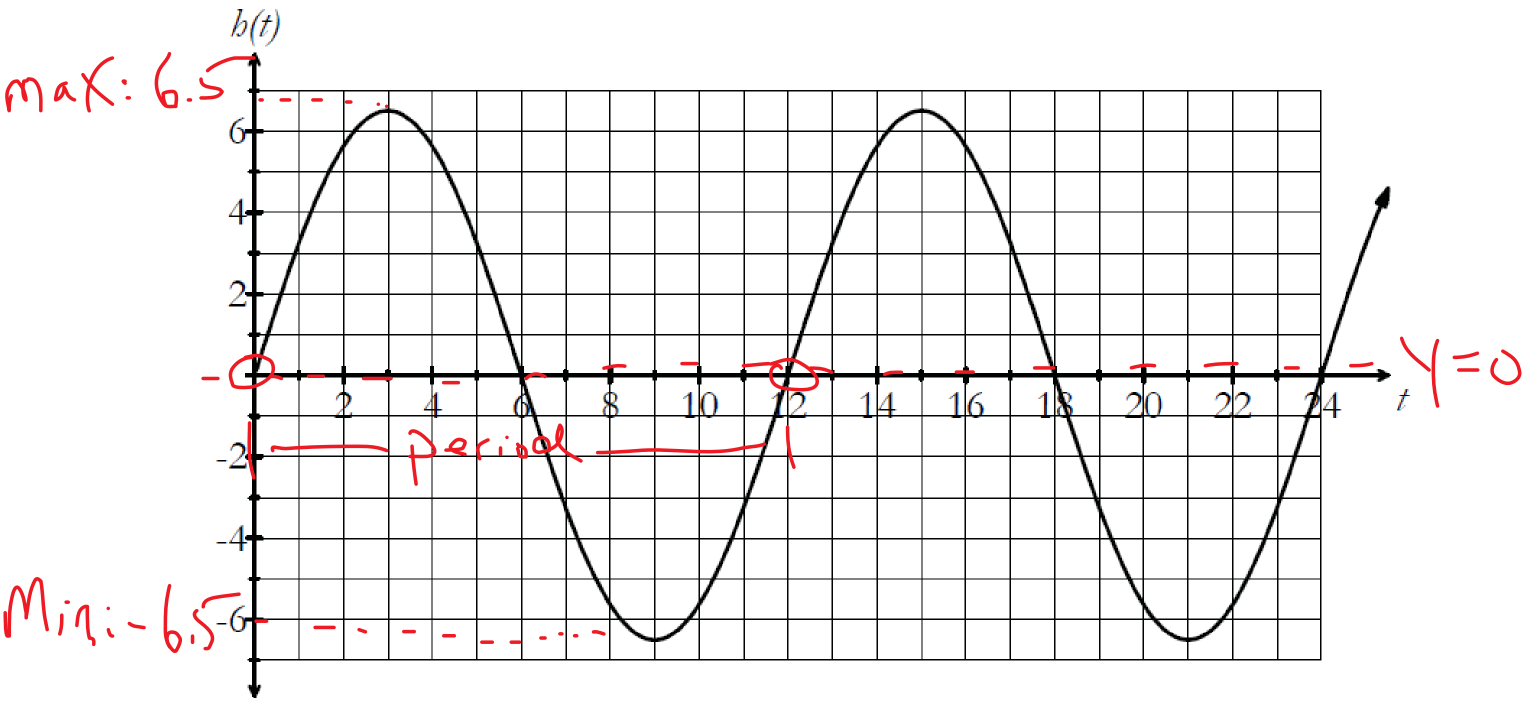
• spinning in the opposite direction

or

• different starting point



Example 3:
The following graph represents the rise and fall of sea level in part of the Bay of Fundy, where t is the time, in hours, and $h(t)$ represents the height relative to the mean sea level:



(A) What is the range of the tide levels?

Range: $\{y \mid -6.5 \leq y \leq 6.5, y \in \mathbb{R}\}$

(B) What does the equation of the midline represent in the graph?

$y=0$ Half tide.

(C) What is the period of the graph?

12 hours

(D) The equation of the sinusoidal function is represented by $h(t) = 6.5\sin\frac{\pi}{6}t$. Calculate the period from the equation and compare it to your answer in ~~(ii)~~. (C)

$$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{\frac{\pi}{6}} = 2\cancel{\pi} \cdot \frac{6}{\cancel{\pi}} = 12$$

Same as (C).

Example 4:
The temperature of an air-conditioned home on a hot day can be modelled using the function $t(x) = 1.5(cos15^o x) + 20$, where x is the time in minutes after the air conditioner turns on and $t(x)$ is the temperature in degrees Celsius. Ask students to answer the following:

(A) What are the maximum and minimum temperatures in the home?

$a = 1.5$

$d = 20$

$\max = d + a = 20 + 1.5 = 21.5°C$

$\min = d - a = 20 - 1.5 = 18.5°C$

(B) What is the temperature 10 minutes after the air conditioner has been turned on?

$X = 10$

$t(x) = 1.5[\cos 15(10)] + 20$

$t(x) = 18.7°C$

(C) What is the period of the function? How would you interpret this value in this context?

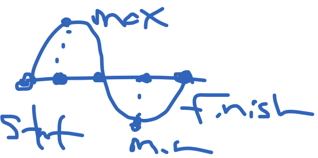
$$\text{period} = \frac{360^o}{b} = \frac{360^o}{15^o} = 24 \text{ minutes}$$

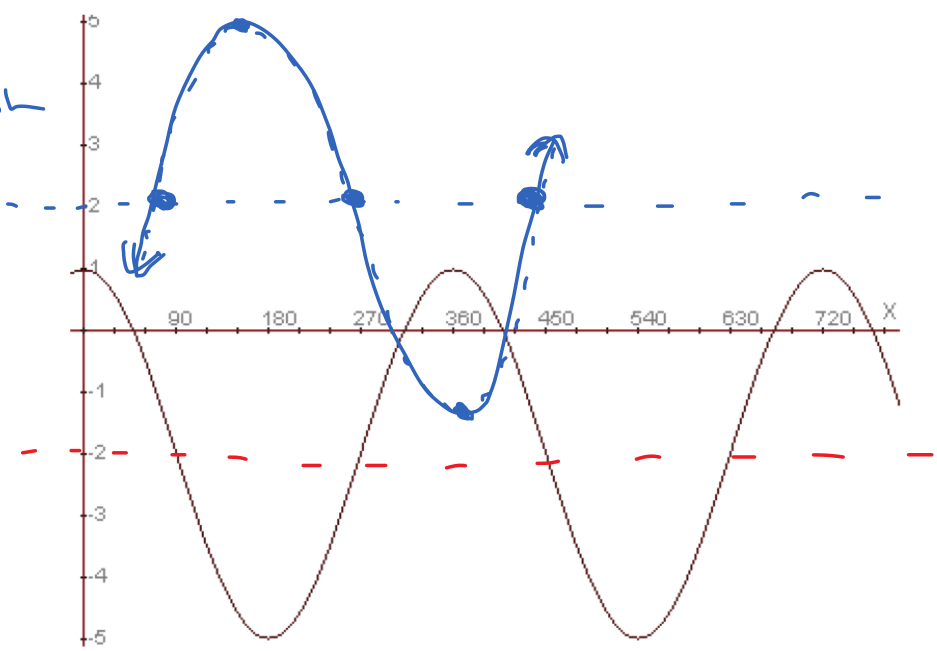
It takes 24 minutes for the house to cool down and warm back up again.

Example 5:

Ashley created the following graph for the equation $y = 3\sin(x - 90^o) + 2$ as shown below.

a b c d





a: 3
b: 1 3rd
c: 90° 2nd
d: 2 1st

$\text{period} = \frac{360^\circ}{1} = 360^\circ$

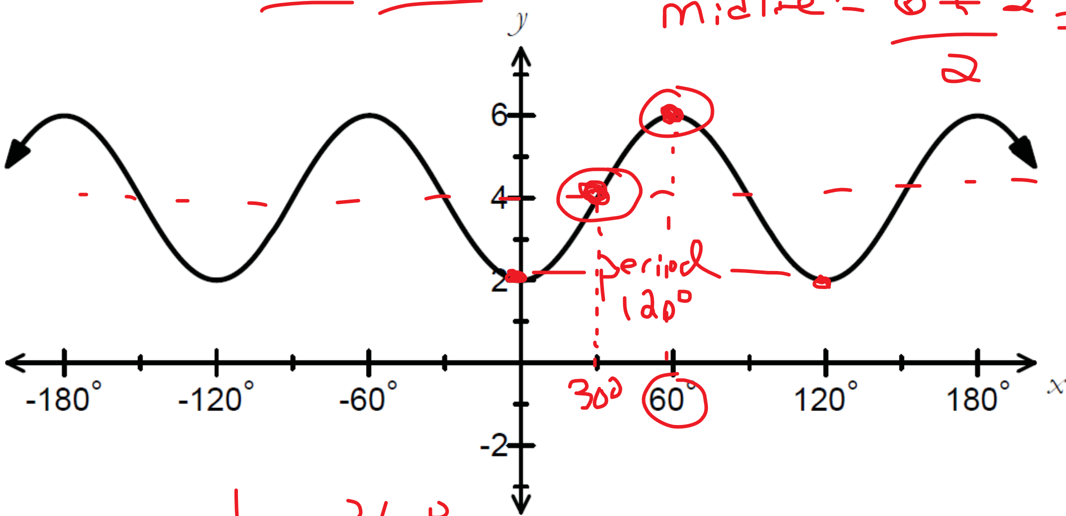
Identify the error Ashley made and then construct the correct graph.

- Midline
- graph is inverted

Example 6: #Public

What is the equation of the sinusoidal function that models the following graph? Express the function in terms of sine and cosine.

$\text{midline} = \frac{6+2}{2} = 4$



a: 2
b: 3
c: sin 30°
cos 60°
d: 4

$$b = \frac{360°}{\text{period}} = \frac{360°}{120} = 3$$

$$y = a\sin b(x-c) + d$$

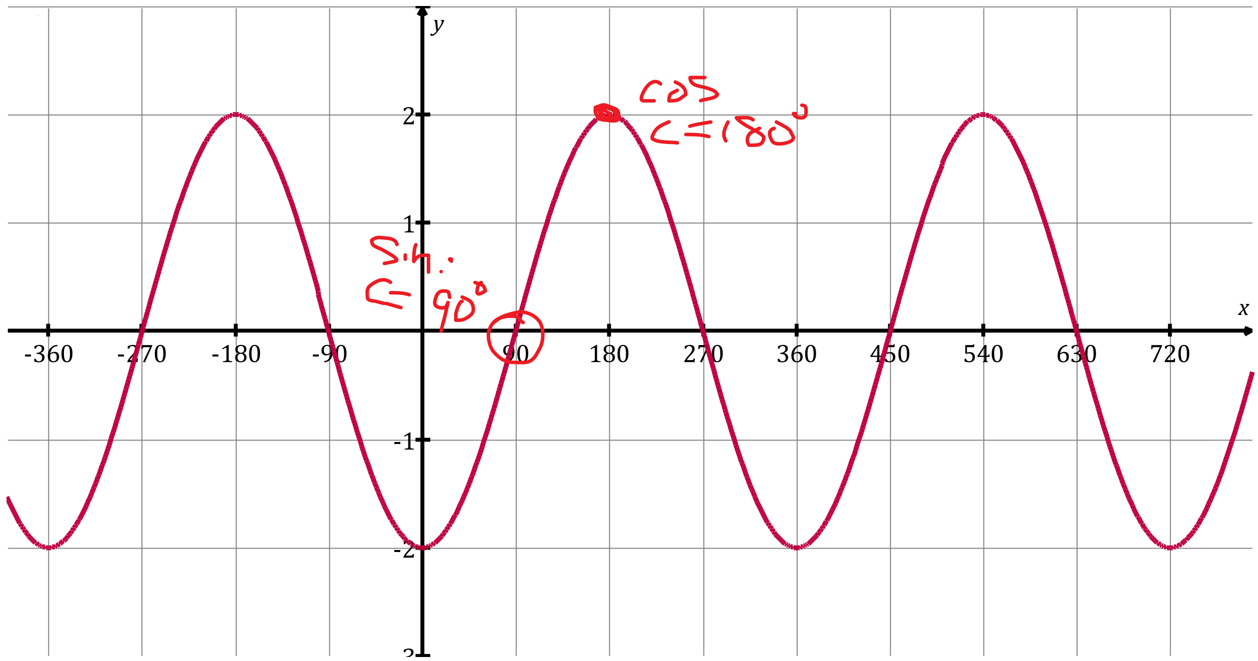
or

$$y = a\cos b(x-c) + d$$

$$y = 2\sin 3(x - 30°) + 4$$

$$y = 2\cos 3(x - 60°) + 4$$

Example 7:
Consider the following graph:



(A) Identify the midline, amplitude, period and range of the graph.

midline: $y=0$

amplitude: 2

period $=360°$

Range: $\{y \mid -2 \le y \le 2, y \in R\}$

(B) Identify the a, b, c, and d values of this equation if it is a sine function. and cosine.

$a=2$

$b=\frac{360°}{360°}=1$

c: sin 90°

cos 180°

$d=0$

(C) What is the equation of the sinusoidal function in terms of sine and cosine?

$y=2\sin(x-90°)$

$y=2\cos(x-180°)$

Example 8:
A graphing calculator is used to carry out a sinusoidal regression on a set of data, and the following information is obtained.

SinReg
y=a*sin(bx+c)+d
a=10.45637298787
b=-0.672234657463
c=-3.563783283744
d=20.45463473738

*Radian Mode

(A) Write the sinusoidal equation for the function. Hint: follow the format of the equation given in the box – it's slightly different than the format we normallyuse.

$y = 10.5\sin(-0.7x - 3.6) + 20.5$

(B) Determine the value of y when $x = 10$.

$y = 10.5\sin[-0.7(10) - 3.6] + 20.5 = 30.2$

(C) Determine the maximum and minimum values of the function.

Max: $d + a = 20.5 + 10.5 = 31.0$

Min: $d - a = 20.5 - 10.5 = 10.0$

Textbook Questions: page 529 - 531 #12, 18, 19, 20