

## 9.1 Financial Mathematics: Borrowing Money

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### Simple vs. Compound Interest

**Simple Interest:** the amount of interest that you pay on a loan is calculated ONLY based on the amount of money that you borrow. That is, you only pay interest on the money that you borrow.

**Compound Interest:** interest is paid on two different things:  
(i) the amount of money that you borrow and  
(ii) the interest that you pay on the money that you borrowed.

When we talk about borrowing money, there are two groups of people involved:

1. The **customer** is the person/group who borrows the money.
2. The **lender** is the the person/institution who loans out the money. It may be a friend, a family member, bank, etc.

Different groups benefit differently from various types of interest. For example, a compound rate involves a greater amount of interest being paid than a simple rate. The lender, or person/group who loans out money would benefit more from this, since they will earn more money in interest. The customer, or person/group who borrows the money is disadvantaged by this however because it means they will pay out more in interest.

There are two main factors that affect the advantages and disadvantages of different types of interest:

1. Whether you are a **customer or lender**: ie. compound interest is an advantage for a lender, but a disadvantage for a customer, as discussed previously.
2. Whether you are **borrowing or investing** money: simple interest would be better for a customer who is borrowing money since it results in the customer having to pay out less money in interest. However, compound interest is better for a customer who is investing money since it means they will gain more money on their investment through interest.

When are simple and compound interest rates typically used?

**Simple Interest:**

- loans from family members or friends
- loans or investments of a year or less

**Compound Interest:**

- Most products offered by financial institutions. See table below.

Borrowing	Investing
Loan	Savings Account
Credit Card	Chequing Account
Mortgage	GIC
Line of Credit	Canada Savings Bond
Student Loan	

**\*Exception:** GICs and Canada Savings Bonds can have either simple or compound interest.

**Calculations Involving Simple and Compound Interest**

**Simple Interest Formula**

$$A = P + Prt$$

where

- $A$  represents the amount present
- $P$  represents the principal amount
- $r$  = interest percentage divided by 100
- $t$  represents the number of years

We can factor the  $P$  out of the right hand side of the equation to give:

$$A = P(1 + rt)$$

**Compound Interest Formula:**

$$A = P(1 + i)^n$$

where

- $P$  is the the principle amount
- $i$  is the interest rate per compounding period
- $n$  is the number of compounding periods

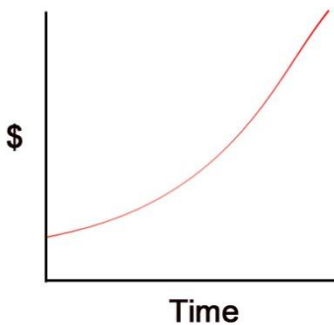
Notice that  $i$  is the interest rate **per compounding period**. If interest is compounded  $x$  times each year, then the given percentage must be divided by  $x$  to come up with  $i$ .

Compounding periods are usually daily, weekly, semi-monthly, monthly, quarterly, semi-annually or annually. The table below shows how many times interest is paid, and the interest rate for each of these options.

Compounding Period	Number of Times Interest Is Paid	Interest Rate per Compounding Period, $i$
daily	365 times per year	$i = \frac{\text{annual rate}}{365}$
weekly	52 times per year	$i = \frac{\text{annual rate}}{52}$
semi-monthly	24 times per year	$i = \frac{\text{annual rate}}{24}$
monthly	12 times per year	$i = \frac{\text{annual rate}}{12}$
quarterly	4 times per year	$i = \frac{\text{annual rate}}{4}$
semi-annually	2 times per year	$i = \frac{\text{annual rate}}{2}$
annually	1 time per year	$i = \frac{\text{annual rate}}{1}$

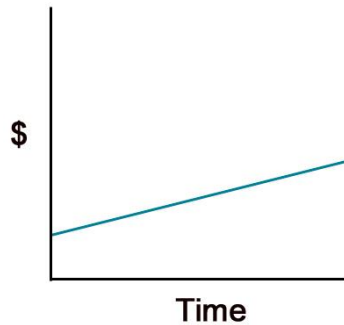
**Compound interest** increases ~~linearly~~ over time.

*exponential*

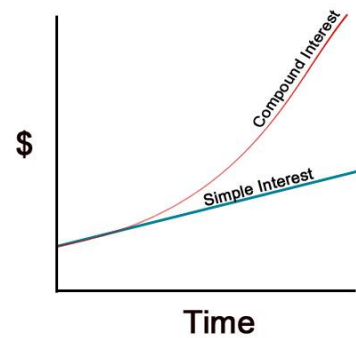


**Simple interest** increases ~~exponentially~~ over time.

*linear*



Both types graphed on the same axis.



**Example 1:**

James intends to go to university in five years. His grandmother decides to invest \$2000 in a Guaranteed Investment Certificate (GIC) to help with his first-year expenses.

(A) How much would the GIC be worth in 5 years if she chooses a simple interest GIC at 3% annual interest?

$$\begin{aligned}
 P &= 2000 & A &= P(1 + rt) \\
 r &= 0.03 & &= 2000 [1 + 0.03(5)] \\
 t &= 5 & &= 2000(1.15) \\
 & & &= 2300
 \end{aligned}$$

$$I_{\text{Interest}}: \$2300 - 2000 = \$300$$

(B) How much would it be worth if the interest is compounded monthly?

$$\begin{aligned}
 P &= 2000 & A &= P(1 + i)^n \\
 i &= \frac{0.03}{12} = 0.0025 & &= 2000(1 + 0.0025)^{60} \\
 n &= 12 \times 5 & &= 2000(1.0025)^{60} \\
 &= 60 & &= 2323.23
 \end{aligned}$$

$$I_{\text{Interest}}: \$2323.23 - 2000 = \$323.23$$

(C) Which option is better for the bank?

Simple. They pay out less interest.

(D) Which is better for James?

Compound. James makes more money.

**Example 2:**

Suppose Peter borrows \$1000 from his parents.

(A) How much will he have to pay back in 2 years if they charge 3% simple interest per year?

$$P = 1000$$
$$r = 0.03$$
$$t = 2$$

$$A = P(1 + rt)$$
$$A = 1000 [1 + 0.03(2)]$$
$$A = 1000(1.06)$$
$$A = \$1060.00$$

(B) How much will you have to pay back in 2 years if they charge 3% interest compounded monthly?

$$P = 1000$$
$$i = \frac{0.03}{12} = 0.0025$$

$$A = 1000(1 + 0.0025)^{24}$$
$$A = 1000(1.0025)^{24}$$
$$A = \$1061.76$$

$$n = 12 \times 2 = 24$$

(C) Explain which is the better option for him.

Simple. He pays less interest.

**Example 3:**

Predict which of the following investments would yield the greater return:

Option 1: \$1000 at 3.5% annual simple interest

Option 2: \$1000 at 3% annual compound interest

Verify your prediction by calculating the value of the investments after 5 years.

① Simple

$$\begin{aligned} A &= 1000 [1 + 0.035(5)] \\ &= 1000(1 + 0.175) \\ &= 1000(1.175) \\ &= \$1175.00 \end{aligned}$$

② Compound <sup>1x5</sup>

$$\begin{aligned} A &= 1000 \left(1 + \frac{0.03}{1}\right)^{5 \cdot 1} \\ A &= 1000(1.03)^5 \\ A &= \$1159.27 \end{aligned}$$