Math 3201

9.2A Paying Back Loans

A loan can involve **regular loan payments** over the term of the loan or a **single payment** at the end of the term.

We will consider two cases:

1. A loan is paid off using a single payment at the end of the term.
2. A loan is paid off by making regular loan payments only cases in which payment frequency matches the compounding period.

We will start off by looking at loans that are paid off using a single payment at the end of the term.

Examples:

- a farmer making a single lump sum payment on his loan after his crop has been harvested
- a payday loan offered by certain financial service providers

Single Loan Payments

For single loan payments when the loan is paid off in full at the end of the term, we can use the formulas from lesson 1 to determine things such as amount and interest.

Most commonly, there will be a compound interest rate, so we can use the formula:

$$A = P(1 + i)^n$$

We could also create a table to show the amount and the interest at the end of each year.

Example 1:

Shannon's employer loaned her $10000 at a fixed rate of 6%, compounded annually, to pay for college tuition and textbooks. The loan is to be repaid in full in a single payment on the maturity date, which is at the end of 5 years.

(A) Complete the following table to show the amount owed at the end of each year, along the interest accumulated.

End of Year	Amount ($)	Interest ($)
0		
1		
2		
3		
4		
5		

(B) Use the compound interest formula to determine how much Shannon owes her employer at the end of the 5 years.

$P = 10000$

$i = \frac{0.06}{1}$

$n = 5 \times 1 = 5$

$A = P(1+i)^n$

$A = 10000(1.06)^5$

$A = \$13382.26$

(C) How much interest does she pay on the loan?

$$\begin{array}{r} \$13382.26 \\ -\ 10000.00 \\ \hline \$3382.26 \end{array}$$

(D) Calculate the amount that she would ~~own~~ owe after 5 years if interest was compounded monthly instead.

$P = 10000$

$i = \frac{0.06}{12} = 0.005$

$n = 5 \times 12 = 60$

$A = 10000(1.005)^{60}$

$= \$13488.50$

(E) How much interest would she pay in this case?

$$\begin{array}{r} \$13488.50 \\ -\ 10000.00 \\ \hline \$3488.50 \end{array}$$

* More compounding periods = more interest

Example 2:

Mary borrows $1000 at 10% interest, compounded semiannually. Sean borrows $1000 at 10% interest compounded annually. How much interest will each pay at the end of two years?

2

1

Mary

$P = 1000$

$i = \frac{0.10}{2} = 0.05$

$n = 2 \times 2 = 4$

$A = 1000(1.05)^4$

$A = \$1215.51$

Interest: $1215.51
− 1000.00
$215.51

Sean

$P = 1000$

$i = \frac{0.10}{1} = 0.10$

$n = 2 \times 1 = 2$

$A = 1000(1.10)^2$

$A = \$1210.00$

Interest: $1210.00
− 1000.00
$210.00

In the previous examples, the principal amount, the interest rate and the term of the loan were kept constant, but the number of compounding periods changed.

What impact does the number of compounding periods have on the total amount of interest paid on a loan?

More Compounding periods = more interest!!

When making financial decisions, it is important for students to understand the **rate of interest charged**, as well as the **compounding**, as these can create large differences over long periods of time.

Example 3:
Which represents the lowest interest that would be paid?

(A) 10% compounded daily

(B) 10% compounded monthly

(C) 10% compounded annually

(A) $i = \frac{0.10}{365} = 0.00027$ $A = P(1.00027)^{365}$

$A = 1.105P$

(B) $i = \frac{0.10}{12} = 0.0083$ $A = P(1.0083)^{12}$

$A = 1.104P$

(C) $i = \frac{0.10}{1} = 0.10$

$A = P(1.10)^1$

$A = 1.10P$

lowest interest

Example 4:
Which represents the lowest interest that would be paid?

(A) 8% compounded daily lowest.

(B) 12% compounded monthly

(A) $i = \frac{0.08}{365} = 0.000219$

$A = P(1.000219)^{365}$
$A = 1.083P$ (8.3%)

(B) $i = \frac{0.12}{12} = 0.01$

$A = P(1.01)^{12}$
$A = 1.127P$ (12.7%)

Determining Interest Rates from A Compound Interest Equation

Example 5:
A loan has interest that is compounded annually. The amount is represented by $A = 1000(1.06)^5$. What is the interest rate?

$1 + i = 1.06$
$i = 1.06 - 1$
$i = 0.06$

$0.06 \times \underset{\text{compounding}}{1} = 0.06$

interest rate

Calculating the Principal Amount When the Amount Repaid is Known

Example 6:
A student repaid a total of $8456.65 including both the principal and interest. If the interest rate was 3% compounded quarterly for 5 years, what was the principal amount of the loan?

$A = 8456.65$

$i = \frac{0.03}{4} = 0.0075$

$n = 5 \times 4 = 20$

$A = P(1+i)^n$

$8456.65 = P(1.0075)^{20}$

$\frac{8456.65}{1.161184} = \frac{1.161184P}{1.161184}$

$P = \$7282.78$

Your Turn:

1. Which situation would result in the greatest interest being paid?
   (A) 8% compounded monthly

   (B) 6% compounded annually

   (C) 6% compounded semiannually

(A) $A = P\left(1+\frac{0.08}{12}\right)^{12} = P(1.0067)^{12} = 1.083P$

(B) $A = P\left(1+\frac{0.06}{1}\right)^{1} = P(1.06)^{1} = 1.060P$

(C) $A = P\left(1+\frac{0.06}{2}\right)^{2} = P(1.03)^{2} = 1.061P$

greatest interest

2. A student repaid a total of \$6532.45 including both the principal and interest. If the interest rate was 5% compounded quarterly for 6 years, what was the principal amount of the loan?

$A = 6532.45$

$i = \frac{0.05}{4} = 0.0125$

$n = 6 \times 4 = 24$

$A = P(1+i)^n$

$6532.45 = P(1.0125)^{24}$

$\frac{6532.45}{1.347} = \frac{1.347P}{1.347}$

$P = \$4849.62$

3. A loan has interest that is compounded monthly. The amount is represented by $A = 6400(1.00417)^{60}$. What is the interest rate?

$1 + i = 1.00417$

$i = 1.00417 - 1$

$i = 0.00417$

0.00417×12

$= 0.05$

or 5%

4. Jill borrows \$850 at 9% interest, compounded semiannually. Jack borrows \$925 at 9% interest compounded annually. How much interest will each pay at the end of four years?

Jill

$P = 850$

$i = \frac{0.09}{2} = 0.045$

$n = 4 \times 2 = 8$

$A = 850(1.045)^8$

$A = \$1208.79$

$- 850.00$

$\$358.79$

Jack

$P = 925$

$i = \frac{0.09}{1} = 0.09$

$n = 4 \times 1 = 4$

$A = 925(1.09)^4$

$A = \$1305.71$

$- 925.00$

$\$380.71$