Math 3201

9.2B Paying Back Loans

Regular/Multiple Loan Payments
For many types of loans, regular loan payments are made at certain time intervals. This is common for mortgages and vehicle loans.

There are three common types of regular payment schedules:

- Biweekly
- Semi Monthly
- Accelerated Biweekly

To see the differences between these, we will consider an example from the Curriculum Guide in a loan payment of $600 per month is made.

The amount paid each year will vary depending on which type of payment plan you choose. The $600 per month payment will work out as follows:

- Biweekly → $600 \times 12 \div 26 = \$276.92$
  paid 26 times per year
- Semi Monthly → $600 \div 2 = \$300.00$,
  paid 24 times per year (1^{st} and 15^{th} of each month)
- Accelerated Biweekly → $600 \div 2 = \$300.00$
  paid 26 times per year

For a loan that has a $600 per month payment, determine how much will be paid out at 3 years, using each of the three payment options.

Biweekly

$\$276.92 \times 26 \times 3 = \$21599.76$

Semi-Monthly

$\$300 \times 24 \times 3 = \$21600.00$

Accelerated Bi-weekly

$\$300 \times 26 \times 3 = \$23400.00$

Example 1:

(A) 130 biweekly payments are required to pay off a loan. How many years does this represent?

$\frac{130}{26} = 5$ years

(B) 288 semi monthly payments are required to pay off a loan. How many years does this represent?

$\frac{288}{24} = 12$ years

(C) 390 accelerated biweekly payments are required to pay off a loan. How many years does this represent?

$\frac{390}{26} = 15$ years

When people purchase a vehicle, they often link their loan payment schedule to their payroll schedule. Why is this the case?

When it comes to regular loan payments, we will only examine situations in which the **payment frequency matches the compounding period** For example, if the interest is compounded monthly, then the loan repayment occurs monthly.

The formulas that we learned previously apply **only** to **single loan payments at the end of a term**, and thus **cannot** be used in situations in which there is a regular loan payment. What we will do for these types of questions is refer to a table which shows the payment, interest principal and balance.

*Potential Public Question

Example 2:
Mark is buying an ATV for the summer. The bank offers him a loan of $7499.99 to pay for his ATV with an interest rate of 4.5% compounding monthly. If Mark makes 36 monthly payments of $223.10, calculate the total interest paid at the end of the loan.

$\frac{0.045}{12} = 0.00375$

(A) Complete the first three rows of the table:

Month	Payment	Interest	Principal	Balance
				7,499.99
1	223.10	7499.99 × 0.00375 = 28.12	223.10 − 28.12 = 194.98	7499.99 − 194.98 = $7305.01
2	223.10	7305.01 × 0.00375 = 27.39	223.10 − 27.39 = 195.71	7305.01 − 195.71 = $7109.30
3	223.10	7109.30 × 0.00375 = 26.66	223.10 − 26.66 = 196.44	7109.30 − 196.44 = $6912.86

The entire table, as created using software, is shown below:

#/Year	Date	Payment	Interest	Principal	Balance
Loan:	12/01/12				7,499.99
1/01	01/01/13	223.10	28.12	194.98	7,305.01
2/01	02/01/13	223.10	27.39	195.71	7,109.30
3/01	03/01/13	223.10	26.66	196.44	6,912.86
4/01	04/01/13	223.10	25.92	197.18	6,715.68
5/01	05/01/13	223.10	25.18	197.92	6,517.76
6/01	06/01/13	223.10	24.44	198.66	6,319.10
7/01	07/01/13	223.10	23.70	199.40	6,119.70
8/01	08/01/13	223.10	22.95	200.15	5,919.55
9/01	09/01/13	223.10	22.20	200.90	5,718.65
10/01	10/01/13	223.10	21.44	201.66	5,516.99
11/01	11/01/13	223.10	20.69	202.41	5,314.58
12/01	12/01/13	223.10	19.93	203.17	5,111.41
13/02	01/01/14	223.10	19.17	203.93	4,907.48
14/02	02/01/14	223.10	18.40	204.70	4,702.78
15/02	03/01/14	223.10	17.64	205.46	4,497.32
16/02	04/01/14	223.10	16.86	206.24	4,291.08
17/02	05/01/14	223.10	16.09	207.01	4,084.07
18/02	06/01/14	223.10	15.32	207.78	3,876.29
19/02	07/01/14	223.10	14.54	208.56	3,667.73
20/02	08/01/14	223.10	13.75	209.35	3,458.38
21/02	09/01/14	223.10	12.97	210.13	3,248.25
22/02	10/01/14	223.10	12.18	210.92	3,037.33
23/02	11/01/14	223.10	11.39	211.71	2,825.62
24/02	12/01/14	223.10	10.60	212.50	2,613.12
25/03	01/01/15	223.10	9.80	213.30	2,399.82
26/03	02/01/15	223.10	9.00	214.10	2,185.72
27/03	03/01/15	223.10	8.20	214.90	1,970.82
28/03	04/01/15	223.10	7.39	215.71	1,755.11
29/03	05/01/15	223.10	6.58	216.52	1,538.59
30/03	06/01/15	223.10	5.77	217.33	1,321.26
31/03	07/01/15	223.10	4.95	218.15	1,103.11
32/03	08/01/15	223.10	4.14	218.96	884.15
33/03	09/01/15	223.10	3.32	219.78	664.37
34/03	10/01/15	223.10	2.49	220.61	443.76
35/03	11/01/15	223.10	1.66	221.44	222.32
36/03	12/01/15	223.15	0.83	222.32	0.00
	12/31/15	8,031.65	531.66	7,499.99	

Relationships from the Table

To determine the amount of interest paid each month, we need to determine the i-value. Then we multiply the balance from the previous pay period by the i-value.

$$\text{Interest Paid} = i \times \text{Balance during last payment period}$$

When you make a monthly payment, so much of the payment goes down on the loan amount itself, called the principal, while the rest is paid out as interest.

$$\text{Monthly Payment} = \text{Interest Paid} + \text{Principal Paid}$$

We can rearrange the previous equation to determine how much gets paid on the principal once we figure how much goes on interest.

$$\text{Monthly Payment Interest} - \text{Paid} = \text{Principal Paid}$$

The balance remaining on a loan can be calculated each month by taking the previous balance and subtracting the principal paid during that payment period. Interest paid has no effect on the balance remaining. It is only the principal paid each payment period that will decrease the overall amount left owing on the loan.

$$\text{Current Balance} = \text{Previous Balance Principal} - \text{Paid during Current Payment Period}$$

Your Turn: Homework:

1. Lorna is buying a car second hand through a private sale. The bank offers her a loan of $5500 to pay for her car with an interest rate of 4.2% compounding monthly. Lorna makes monthly payments of $210.45. Complete the following table.

Month	Payment	Interest	Principal	Balance
$i = \frac{0.042}{12} = 0.0035$				5500
1	210.45	$5500 \times 0.0035 = 19.25$	$210.45 - 19.25 = 191.20$	$5500 - 191.20 = \$5308.80$
2	210.45	18.58	191.87	$5116.93
3	210.45	17.91	192.54	$4924.39
4	210.45	17.24	193.21	$4731.18

2. The interest on the loan shown in the chart below is 5% compounded monthly. How much of the second payment is the interest toward the loan?

Payment Period (month)	Payment ($)	Principal Paid ($)	Balance ($)
0			15,000
1	450	387.50	14,612.50
2	450	389.11	14,223.39
3	450	390.74	13,832.65

$450 − $389.11 = $60.89