Math 2201 **Maximum/Minimum Problems Practice**

1. On a forward somersault , Greg's height above the water is given by $h = -5t^2 + 6t + 3$, where t is time in seconds and h is height in meters.

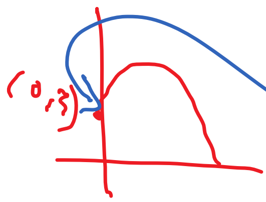
(A) Find Greg's maximum height above the water.

$k = -5(0.6)^2 + 6(0.6) + 3 = 4.8$ m

(B) How long does it take him to reach that maximum height?

$h = -\frac{b}{2a} = -\frac{6}{2(-5)} = \frac{-6}{-10} = 0.6$ s

(C) How high is the diving board?



$h = -5t^2 + 6t + 3$

(D) What is his height after 1.5 seconds?

$h(1.5) = -5(1.5)^2 + 6(1.5) + 3 = 0.75$ m.

2. The power P watts supplied to a circuit by a 9 volt battery is given by the formula $P = 9I - 0.5I^2$ where I is the current in amperes.

$P = -0.5I^2 + 9I$

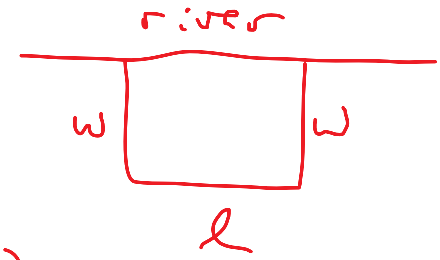
(A) For what value of the current will the power be a maximum?

$h = -\frac{b}{2a} = \frac{-9}{2(-0.5)} = 9$ amps

(B)What is the maximum power?

$k = 9(9) - 0.5(9)^2 = 40.5$ watts.

3. A rectangular lot is bounded on one side by a river and on the other three sides by 80 m of fencing. Find the dimensions that will enclose the maximum area.



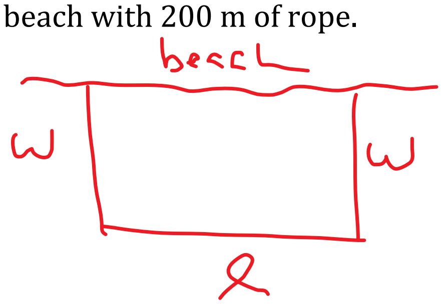
$$l + 2w = 80$$
$$A = l \cdot w$$
$$l = -2w + 80$$
$$A = (-2w + 80)w$$
$$A = -2w^2 + 80w$$
$$h = \frac{-b}{2a} = \frac{-80}{2(-2)} = 20$$

$w = 20\text{m}$

$l = -2(20) + 80 = 40\text{m}$

$A = -2(20)^2 + 80(20) = 800\text{m}^2$

4. A lifeguard marks off a rectangular swimming area at a beach with 200 m of rope. What is the greatest area she can enclose?



$$l + 2w = 200$$
$$A = l \cdot w$$
$$l = -2w + 200$$
$$A = (-2w + 200)w$$
$$A = -2w^2 + 200w$$
$$h = \frac{-b}{2a} = \frac{-200}{2(-2)} = 50$$

$w = 50\text{m}$

$l = -2(50) + 200 = 100\text{m}$

$A = -2(50)^2 + 200(50) = 5000\text{m}^2$

or

$A = l \cdot w = (50)(100) = 5000\text{m}^2$

5. 80 m of fencing are available to enclose a rectangular play area. What dimensions will yield the maximum area? What is the maximum area?

$2l + 2w = 80$

$A = l \cdot w$

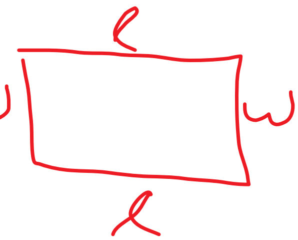
$\frac{2w}{2} = \frac{-2l + 80}{2}$

$w = -l + 40$

$A = l(-l + 40)$

$A = -l^2 + 40l$

$h = \frac{-b}{2a} = \frac{-40}{2(-1)} = 20$



$l = 20\text{m}$

$w = -20 + 40 = 20\text{m}$

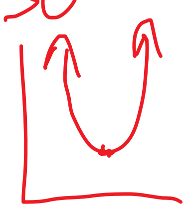
$A = -(20)^2 + 40(l) = 400\text{m}^2$

or

$A = l \cdot w = (20)(20) = 400\text{m}^2$

6. A producer of synfuel from coal estimates that the cost C dollars per barrel for a production run of x thousand barrels is given by $C = 9x^2 - 180x + 940$. How many thousand barrels should be produced each run to keep the cost per barrel at a minimum? What is the minimum cost per barrel of synfuel?

$a > 0$



$h = \frac{-b}{2a} = \frac{-(-180)}{2(9)} = \frac{180}{18} = 10$

$\therefore$ 10 000 barrels

$k = 9(10)^2 - 180(10) + 940 = 40$

\$40/barrel