Math 2201 Unit 6: Quadratic Functions
Summary of Standard Form: $\quad y=a x^{2}+b x+c$

|  | In General $y=a x^{2}+b x+c$ | Example 1 $y=-3 x^{2}+3 x+10$ | $\begin{array}{r} \text { Example } 2 \\ y=2 x^{2}+12 x-13 \end{array}$ |
| :---: | :---: | :---: | :---: |
| Direction of Opening | $a>0$, graph opens up $a<0$, graph opens down | $a=-3$, graph opens down | $a=2$, graph opens up |
| Y-Intercept | c-value | $\mathrm{c}=10$, so $y$-int. $=10$ or $(0,10)$ | $\mathrm{c}=-13$, so $y$-int. $=-13$ or (0,-13) |
| Equation for axis of Symmetry | $x=\frac{-b}{2 a}$ | $\begin{aligned} \mathrm{a}=-3, \mathrm{~b}=3, & \mathrm{c}=10 \\ x & =\frac{-b}{2 a} \\ x & =\frac{-3}{2(-3)} \\ x= & \frac{1}{2} \text { or } 0.5 \end{aligned}$ | $\begin{aligned} \mathrm{a}=2, \mathrm{~b}=12, \mathrm{c} & =-13 \\ x & =\frac{-b}{2 a} \\ x & =\frac{-12}{2(2)} \\ x & =-3 \end{aligned}$ |
| Vertex | $x$-value is the same as the axis of symmetry. To find $y$ coordinate, plug $x$-value into equation. | $\begin{aligned} & x=0.5 \text { (see above) } \\ & y=-3 x^{2}+3 x+10 \\ & y=-3(0.5)^{2}+3(0.5)+10 \\ & y=10.75 \\ & \text { Vertex: } \quad(-0.5,10.75) \\ & \hline \end{aligned}$ | $\begin{aligned} & x=-3(\text { see above }) \\ & y=2 x^{2}+12 x-13 \\ & y=2(-3)^{2}+12(-3)-13 \\ & y=-31 \\ & \text { Vertex: }(-3,-31) \end{aligned}$ |
| Maximum/Minimum Value | When the graph opens down, the $y$-coordinate of the vertex is called the max. value. <br> When the graph opens up, the $y$-coordinate of the vertex is called the min. value. | Since the graph opens down, it has a maximum value, and the value is 10.75 | Since the graph opens up, it has a minimum value, and the value is -31 |
| Range | ```Opens Down \(\{y \mid y \leq y-\) value of vertex, \(y \in R\}\) Opens Up \(\{y \mid y \geq y-\) value of vertex, \(y \in R\}\)``` | $\{y \mid y \leq 10.75, y \in R\}$ | $\{y \mid y \geq-31, y \epsilon R\}$ |

## Math 2201 Unit 6: Quadratic Functions

Summary of Factored Form: $\quad y=a(x-r)(x-s)$

|  | In General $y=a(x-r)(x-s)$ | Example 1 $y=2(x+3)(x-4)$ | Example 2 $y=-3 x(x+4)$ |
| :---: | :---: | :---: | :---: |
| Direction of Opening | a $>0$, graph opens up a < 0, graph opens down | $a=2$, opens up | $a=-3$, opens down |
| Y-Intercept | Set $x=0$ and solve for y . | $\begin{aligned} & y=2(0+3)(0-4) \\ & y=-24 \text { or }(0,-24) \end{aligned}$ | $\begin{aligned} & y=-3(0)(0+4) \\ & y=0(4)=0 \text { or }(0,0) \end{aligned}$ |
| X - Intercepts | $\begin{aligned} & \mathrm{x}-\mathrm{r}=0, \text { therefore } \mathrm{x}=\mathrm{r} \\ & \mathrm{x}-\mathrm{s}=0 \text {, therefore } \mathrm{x}=\mathrm{s} \end{aligned}$ | $\begin{array}{ll} x+3=0 & x-4=0 \\ x=-3 & x=4 \\ (-3,0) \text { and }(4,0) & \\ \hline \end{array}$ | $\begin{array}{\|cl} \hline-3 x=0 & x+4=0 \\ x=0 & x=-4 \\ (0,0) \text { and }(-4,0) & \\ \hline \end{array}$ |
| Equation for axis of Symmetry | Take the average of the x intercepts. $x=\frac{r+s}{2}$ | $\begin{gathered} x=\frac{r+s}{2} \\ x=\frac{-3^{2}+4}{2} \\ x=\frac{1}{2} \text { or } 0.5 \end{gathered}$ | $\begin{gathered} x=\frac{r+s}{2} \\ x=\frac{0+(-4)}{2} \\ x=\frac{-4}{2} \text { or }-2 \end{gathered}$ |
| Vertex | $x$-value is the same as the axis of symmetry. To find $y$-coordinate, plug $x$-value into equation. | $\begin{aligned} & x=0.5(\text { see above }) \\ & y=2(0.5+3)(0.5-4) \\ & y=2(3.5)(-3.5) \\ & y=-24.5 \end{aligned}$ $\text { Vertex: }(0.5,-24.5)$ | $\begin{aligned} & x=-2(\text { see above }) \\ & y=-3 x(x+4) \\ & y=-3(-2)[-2+4] \\ & y=6[2]=12 \\ & \text { Vertex: }(-2,12) \end{aligned}$ |
| Maximum/Minimum Value | When the graph opens down, the $y$-coordinate of the vertex is called the max. value. <br> When the graph opens up, the $y$ coordinate of the vertex is called the min. value. | Minimum Value, $\mathrm{y}=-24.5$ | Maximum Value, y = 12 |
| Range | ```Opens Down {y\|y\leqy-value of vertex, y\inR} Opens Up {y|y\geqy-value of vertex, y\inR}``` | $\{y \mid y \geq-24.5, y \in R\}$ | $\{y \mid y \leq 12, y \epsilon R\}$ |

## Math 2201 Unit 6: Quadratic Functions

Summary of Vertex Form: $\quad y=a(x-h)^{\mathbf{2}}+k$

|  | In General $y=a(x-h)^{2}+k$ | $\begin{gathered} \text { Example } 1 \\ y=4(x-2)^{2}+3 \end{gathered}$ | $\begin{gathered} \text { Example } 2 \\ y=-2(x+5)^{2}-6 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| Direction of Opening | a $>0$, graph opens up $a<0$, graph opens down | $a=4$, opens up | $\mathrm{a}=-2$, opens down |
| y - Intercept | Set $\mathrm{x}=0$ and solve for y . | $\begin{aligned} & y=4(x-2)^{2}+3 \\ & y=4(0-2)^{2}+3 \\ & y=4(4)+3=16+3=19 \end{aligned}$ | $\begin{aligned} & y=-2(x+5)^{2}-6 \\ & y=-2(0+5)^{2}-6 \\ & y=-2(25)-6=-50-6=-56 \end{aligned}$ |
| Equation for axis of Symmetry | $x=h$ | $x=2$ | $x=-5$ |
| Vertex | Takes the form ( $\mathrm{h}, \mathrm{k}$ ) | $(\mathrm{h}, \mathrm{k})=(2,3)$ | $(\mathrm{h}, \mathrm{k})=(-5,-6)$ |
| Maximum/Minimum Value | When the graph opens down, the $y$-coordinate of the vertex is called the max. value. <br> When the graph opens up, the $y$ coordinate of the vertex is called the min. value. | Minimum Value, $\mathrm{y}=3$ | Maximum Value, $\mathrm{y}=-6$ |
| Range | Opens Down <br> $\{y \mid y \leq y-$ value of vertex, $y \in R\}$ <br> Opens Up <br> $\{y \mid y \geq y-$ value of vertex, $y \in R\}$ | $\{y \mid y \geq 3, y \in R\}$ | $\{y \mid y \leq-6, y \in R\}$ |

